Ambiguous Policy Announcements

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September 11, 2017

Preliminary and Incomplete

Abstract

We study the effects of an announcement of a future shift in monetary policy in a new Keynesian model, where ambiguity-averse households with heterogeneous net financial wealth use a worst-case criterion to assess the credibility of the announcement. The response of aggregate demand to the announcement of a future loosening in monetary policy falls when financial wealth is more concentrated. The concentration of financial wealth matters because households with great net financial wealth (creditors) are those who are the most likely to believe the announcement, due to the potential loss of wealth from the prospective policy easing. And when creditors believe the announcement more than debtors, their expected wealth losses are larger than the wealth gains that debtors expect. So aggregate net wealth is perceived to fall, which can even lead to a contraction in aggregate demand when financial wealth is concentrated enough. In general, forward guidance announcements may have little or even perverse effects when wealth inequality is large and agents face Knightian uncertainty about the credibility of announcements.

*We thank Manuel Amador, Gene Ambrocio, Philippe Andrade, Francesco Caselli, Martin Ellison, Gaetano Gaballo, Nicola Gennaioli, Luigi Guiso, Javier Suarez, Giovanni Violante, and Mirko Wiederholt for useful comments, as well as seminar participants at ADEMU-CEPR-CfM-ERC Conference at UCL, AEA meetings 2016, Bank of Italy, Bank of Spain, Bocconi University, CREI, CUHK, EIEF, EABCN Meeting 2017, ESSIM 2016, HKUST, First International Conference on Financial Markets and Macroeconomic Performance at Goethe University, Minneapolis FED, Research Meeting on Dynamic Macroeconomics at LUISS, SED 2016 meeting, Summer Workshop in International Finance and Macro Finance at SciencesPo, and UCL. We acknowledge the financial support of the European Research Council under ERC Advanced Grant 293692 and ERC Starting Grant 676846. E-mail: c.michelacci1968@gmail.com, gigipaciello@gmail.com. Postal address: EIEF, Via Sallustiana, 62, 00187 Roma, Italy.

1 Introduction

Policy makers often use announcements of future reform of economic institutions or changes in fiscal or monetary policy to stimulate the economy in the short run. These policies may represent a shift in the established historical objectives of the policy maker and typically carry important redistributive implications. For example during the Great Recession, with nominal short-term interest rates at the zero lower bound, central banks have relied extensively on announcements of future monetary policy changes to raise current inflation and stimulate the economy, a practice generally known as *forward guidance*. These announcements may sometimes appear to be in contrast with the legally stated primary objective of the central bank (price stability) and it is well known that inflation tends to redistribute wealth from creditors to debtors (Fisher 1933, Doepke and Schneider 2006, and Adam and Zhu 2015). In this paper we show that when agents are ambiguity-averse, these policy announcements can have little and sometimes even unintended effects in the period before the new policy is actually implemented. Generally the effect of the announcement depends on (i) the amount of redistribution that the policy change will induce, (ii) the concentration of future hypothetical wealth losses, and (iii) the (endogenous) correlation between agents' wealth and the change in their expectations with the announcement.

We consider the impact of monetary policy announcements in a new Keynesian business cycle model, where sticky prices allow for changes in nominal interest rates to cause changes in real interest rates. We analyze the effects of announcements of future changes in nominal interest rates in an economy where agents have well defined expectations about the future dynamics of the economy in the absence of the announcement, while they face uncertainty about the credibility of the announcement. In this sense the announcements are "ambiguous". Heterogeneity in households' wealth causes heterogeneous exposure of households' income to changes in real interest rates. Ambiguity-averse households use a worst-case criterion in assessing the credibility of announcements, according to the Maximin preference specification proposed by Gilboa and Schmeidler (1989). Households with greater net financial wealth (in brief creditors) are more likely than households with little or negative net financial wealth (debtors) to believe the announcement of a future monetary easing, because their worst-case scenario is that real rates, hence financial income, will actually fall. And if creditors ascribe greater credibility to the announcement than debtors do, the wealth losses they expect to incur are larger than the gains that debtors expect, so that aggregate demand behaves as if expected aggregate net wealth falls. We refer to this fall in expected aggregate net wealth as the forward *misguidance* effect, which generally implies that the response of aggregate demand falls when financial wealth is more concentrated. When financial wealth is concentrated enough, the misguidance effect can be so strong to dominate the intertemporal substitution effect on consumption typically emphasized by the literature, and lead to a contraction in activity due to lack of aggregate demand. Generally, when a policy easing is announced, the real rate expected by creditors is lower than that expected by debtors. This produces a rebalancing in the financial asset positions of households and can even cause credit crunches characterized by zero net supply of financial assets, which happen because households undo their positions in order to be fully insured against future monetary policy changes.

In the case of an announcement of a future monetary policy tightening (a rise in future real rates), debtors are the most likely to take the announcement as credible and for them the increase in future rates reduces consumption through both substitution and income effects. So aggregate consumption and output unambiguously fall. When wealth inequality is sufficiently marked, the fall in output is sharper than it would be in a hypothetical equilibrium in which the announcement is fully believed by all agents.

We study the importance of the misguidance effect for an economy in a liquidity trap. We show that the effect is mitigated but still present when households can trade in real as well as in financial assets and when households do not perceive government bonds as real wealth. When announcing a future monetary policy easing, the fall in expected aggregate net wealth can also be mitigated by policies which redistribute wealth from creditors to debtors at the time of the announcement so as to anticipate to today the redistribution induced by the future monetary easing.

We use the start of forward guidance by the ECB on 4 July 2013 to study the quantitative importance of the misguidance effect.[1] For the effect to be present (i) the central bank should announce a commitment to a future policy and (ii) ambiguity-averse households should doubt about the credibility of the announcement, which are both likely to apply in the specific episode: the announcement was generally perceived as a commitment by the ECB on keeping future interest rates low and there was (and still is) substantial debate on whether a future monetary policy easing would imply a violation of the ECB mandate for price stability.[2] We study the effects of the ECB forward guidance announcement in our

[1] On that date the ECB Governing Council announced that "it expected the key ECB interest rates to remain at present or lower levels for an extended period of time."

[2] The international press generally reported the statement by the Governing Council by saying that "the ECB will commit to keeping interest rates low" (see for example "https://www.ft.com/content/827ca972-e4d5-11e2-875b-00144feabdc0")—even if verbs like "pledge", "vow" and "commit" were not used in the original official statement by the ECB. Indeed, after the announcement, long-term government bond yields and EONIA swap rates fell by 5-10 basis points at maturities between 2 and 4 years (see Coeuré (2013), ECB (2014), and Picault (2017)), while inflation expectations were revised upward (Andrade and Ferroni 2016), which is consistent with the "Odyssean" interpretation that the announcement implied a commitment on a future monetary policy easing. During the crisis the ECB has been often accused of violating her mandate for ensuring price stability, even by the President of the Deutsche Bundesbank, Jens Weidmann, generating doubts about the future functioning of the ECB as evidenced by the pronounced increased dispersion in how much European households trust the ECB, see Guiso, Sapienza, and Zingales (2016).

economy, which we assume is initially in a liquidity trap. We use data from the Household Finance and Consumption Survey (HFCS) and match the entire distribution of European households in terms of net financial wealth and real wealth. To calibrate the amount of uncertainty resulting from the announcement, we use a Difference-in-Differences strategy based on quarterly Italian data. We construct a measure of the inflation expectations of households at a highly disaggregated level and find that in response to the ECB announcement creditor households experienced a relative increase in their inflation expectations, which implies the presence of a misguidance effect. We calibrate the ECB announcement to match the average response of expected inflation—which have been revised upward by around 10 basis points in the Euro area—as well as the increased correlation between the inflation expectations of households and their financial asset position as implied by our Dif-in-Dif estimates. We compare the response of our economy where agents are ambiguity averse and face uncertainty about the credibility of the announcement and the full credibility benchmark where all agents accord full credit to the announcement. We find that in our model the effect of the ECB announcement on output is considerably dampened by comparison with the full credibility benchmark. Under our preferred parametrization, output increases by one percentage point before implementation, against a gain of more than 3% under the benchmark.

The literature. Forward guidance has become a central tool of monetary policy as a result of the Great Recession, because conventional monetary expansion was no longer available, with short-term rates at the zero lower bound. There is a growing literature on optimal monetary policy in a liquidity trap (Eggertsson and Woodford 2003) as well as on the effects of forward guidance (Del Negro, Giannoni, and Patterson 2015, Swanson 2016). For conventional new Keynesian sticky-price models it is a puzzle why forward guidance has been little effective in stimulating the economy and getting it out of the liquidity trap. Some papers have proposed explanations for this puzzle, see Andrade, Gaballo, Mengus, and Mojon (2015), Caballero and Farhi (2014), Kaplan, Moll, and Violante (2016b), McKay, Nakamura, and Steinsson (2015) and Wiederholt (2014). In particular Andrade et al. (2015) also emphasize that forward guidance leads to heterogeneity in beliefs as agents could interpret the announcement of future low interest rates either as bad signal of the state of the economy or as good news of a future monetary easing. In our model there is no disagreement on the interpretation of the announcement, which all agents understand as a commitment on a future policy easing, relatively in line with the European experience.[3] The problem is that agents are ambiguity averse and face uncertainty about the credibility

[3] The US and the European experiences are indeed different: announcements of a future policy easing caused downward revisions in inflation expectations in the US (Campbell, Evans, Fisher, and Justiniano 2012), while inflation expectations were revised upward in Europe (Andrade and Ferroni 2016).

of the announcement. As a result the endogenous changes in expectations of households are affected by their wealth position—which is consistent with our empirical evidence—and the concentration of aggregate wealth becomes a key determinant of the effects of forward guidance. The mechanism explains around two thirds of the forward guidance puzzle in Europe.

At least since Fisher (1933) it has been known that expansionary monetary policy redistributes wealth from creditors to debtors. It has also been observed that such redistribution could expand aggregate demand because agents may differ in marginal propensity to consume out of wealth (as first posited by Tobin, 1982), or in portfolio liquidity or term structure, as in Kaplan, Moll, and Violante (2016a) and Auclert (2015) respectively. However, Doepke, Schneider, and Selezneva (2015) postulate an overlapping-generations model in which this redistribution decreases aggregate consumption. Here we focus on the redistribution of expected wealth induced by news about future policies, which, under ambiguity aversion, is a negative-sum game because the net losers tend to believe the news more strongly than the net winners.

Other papers have shown the relevance of ambiguity aversion to business cycle analysis. Ilut and Schneider (2014) show that shocks to the degree of ambiguity can be an important source of cyclical volatility. Backus, Ferriere, and Zin (2015) examine asset pricing, and Ilut, Krivenko, and Schneider (2016) devise methods suitable for dynamic economies where ambiguity-averse agents endogenously differ in their perception of exogenous shocks, and study the implications for precautionary savings, asset premiums and insurance gains, in stochastic steady states. Here instead we focus on the effects of policy announcements, and more generally news about the future, and how they interact with wealth inequality and redistribution.

Section 2 characterizes the economy. Section 3 studies the effect of monetary policy announcements in a simple case and further discusses the mechanism. Section 4 extends the model, which is calibrated in Section 5 to quantify the effects of forward guidance by the ECB in Section 6. Section 7 studies robustness, and Section 8 concludes. The Appendix contains details on theoretical derivations, data and model computation.

2 The model

In this section we consider an economy in discrete time which we can solve analytically. While stylized, the model is rich enough to combine the relevant features of a conventional new Keynesian model of the business cycle, namely an intertemporal Euler equation determining household demand, and sticky prices characterizing firms supply in the goods market, as well as allowing for ambiguity-averse households with heterogeneous wealth.

This framework is used to analytically characterize how an announcement of changes to the path of real interest rates affects the beliefs of the different households, and how the endogenous relationship between household wealth and beliefs matter for the effect of such announcement on today output. After having characterized the model, in Section 3.4 we discuss some of its assumptions and extensions. In Section 4, we extend the model along dimensions that make an analytical characterization no more possible but that are relevant for a quantitative analysis of the importance of our mechanism.

Overview The economy is populated by a unit mass of households, indexed by $x \in [0, 1]$, who are ambiguity-averse and differ only in net financial wealth, $a_{xt} \in [\underline{a}_t, \bar{a}_t]$, which is invested in one-period bonds paying a real interest rate r_t in period t. There is a unit mass of firms that demand labor to produce intermediate goods sold under monopolistic competition; prices are sticky. The nominal interest rate is adjusted to achieve the inflation target set by a monetary policy authority which has an unambiguous mandate to maintain price stability. The monetary authority has always complied with this mandate, fully stabilizing prices over the years. We focus on the short run response of the economy, when the monetary authority suddenly and unexpectedly announces a future change in the inflation target, which makes households doubt whether the authority will actually deviate from its historical mandate, as announced. Hereafter the convention is that, unless otherwise specified, variables are real—measured in units of the final consumption good.

Households Household $x \in [0, 1]$ is infinitely-lived, with a time-t one-period-ahead subjective discount factor β_t. Her per period preferences over consumption c_{xt} and labor l_{xt} are given by

$$U(c_{xt}, l_{xt}) = \frac{\left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma}, \tag{1}$$

with $\psi_0, \psi > 0$ and $\sigma > 1$. When all households share the same beliefs, these preferences (Greenwood, Hercowitz, and Huffman 1988) guarantee that the economy is characterized by a representative household, which is a canonical benchmark in the new Keynesian literature. Financial markets are incomplete, in that households can only invest in a one-period bond, which, at time t, pays (gross) return r_t per unit invested. Households can borrow freely by going short on the asset. The labor market is perfectly competitive, so households take the wage w_t as given. At each point in time t, household x chooses the triple $\{c_{xt}, l_{xt}, a_{xt+1}\}$ subject to the budget constraint

$$c_{xt} + a_{xt+1} \leq w_t\, l_{xt} + r_t\, a_{xt} + \lambda_t, \tag{2}$$

where a_{xt+1} measures the units invested in bonds at time t, while λ_t denotes (lump sum) government transfers (specified below).

Monetary policy rule The real interest rate realized in period t is given by $r_t = R_{t-1}/\Pi_t$, where R_{t-1} is the nominal interest rate set by the monetary authority at period $t-1$, and $\Pi_t = p_t/p_{t-1}$ is gross inflation realized in period t. In particular, the monetary authority sets R_t according to

$$R_t = \min\left\{1, \frac{1}{\beta_t}\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi}\right\}, \tag{3}$$

where $\phi > 1$, $1/\beta_t$ represents the natural rate of interest, and Π_t^* is the time-t *inflation target*, which we assume is equal to one in steady state, $\bar{\Pi}_t^* = 1$.

Firms The final consumption good is produced by a (representative) competitive firm, which uses a continuum of varieties $i \in [0, 1]$ as inputs according to

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \tag{4}$$

where y_{it} is the amount of variety i used in production. The variety i is produced only by a firm i, which uses a linear-in-labor production function, so that $y_{it} = \ell_{it}$, where ℓ_{it} denotes firm i's demand for labor whose unit cost is w_t. Firm $i \in [0, 1]$ sets the nominal price for its variety p_{it} to maximize expected profits at the beginning of the period, $d_{it} \equiv y_{it}\,(p_{it}/p_t - w_t)$, taking as given the demand schedule by the competitive firm, the aggregate nominal price, p_t, and the wage rate, w_t. We assume firm i chooses its nominal price at time t, p_{it}, after the monetary authority has set the inflation target Π_t^*, but before any time-t policy announcement. Finally, we posit initially that the government owns all the firms in the economy and rebates profits back to households in lump-sum fashion, so that $\lambda_t = \int_0^1 d_{it}\, di$.

Market clearing In equilibrium, output Y_t is equal to aggregate consumption $C_t \equiv \int_0^1 c_{xt}\, dx$, so that $Y_t = C_t$, and labor demand is equal to labor supply, $\int_0^1 \ell_{it} di = \int_0^1 l_{xt} dx$. Since bonds are in zero net supply, clearing the financial market requires that $\int_0^1 a_{x,t} = 0$ at the return $r_t = R_{t-1}/\Pi_t$.

Steady state At $t = 0$, the economy is initially in a steady state with $\beta_t = \beta < 1$, where a monetary authority with an unambiguous mandate for price stability has always set $\Pi_t^* = 1$, and households expect Π_t^* to remain equal to one also in any future t, implying $\bar{r} = \bar{R} = 1/\beta$ and $\bar{\Pi} = 1$, where the upper bar denotes the steady state value of the corresponding quantity.

Policy announcement At $t = 0$ (after firms have set their nominal price), the monetary authority announces that in period $T > 0$, and only at T, the inflation target will deviate from full price stability, implying that $\Pi_T^* = \varepsilon$, and $\Pi_t^* = 1$ for all $t \neq T$. If $\varepsilon > 1$, the announcement is *inflationary*; if $\varepsilon < 1$, it is *deflationary.* On the basis of the announcement, household $x \in [0, 1]$ makes her decisions on consumption, labor supply and saving, while firm $i \in [0, 1]$ supplies any amount demanded at its set price.

Ambiguity aversion and uncertainty Households are ambiguity-averse as in the multiple priors utility model of Gilboa and Schmeidler (1989), whose axiomatic foundations are provided by Epstein and Schneider (2003). Households doubt whether the authority will actually deviate from her mandate for price stability and assume that the monetary authority sets the inflation target at time T, Π_T^*, to minimize

$$\boldsymbol{L} = (1 - \Pi_T^*)^2 + \gamma\,(\epsilon - \Pi_T^*)^2 \tag{5}$$

where $\gamma \in R^+$ measures the *credibility* of the monetary authority and ϵ is the monetary announcement, with the convention that $\epsilon = 1$ denotes no announcement. The first term in (5) is the cost to the authority of deviating from price stability, the second is the credibility cost of reneging the announcement. The credibility parameter γ is fully known to the monetary authority, so households infer that

$$\Pi_T^* = \frac{1 + \gamma\varepsilon}{1 + \gamma}. \tag{6}$$

Households have multiple priors about the probability distribution of γ and we start assuming that γ could be any random variable on the positive real line. Given (6), then households conclude that all values of Π_T^* in the interval

$$\mathcal{S}_{T-1} = [\min\{\varepsilon, 1\}, \max\{\varepsilon, 1\}] \tag{7}$$

are feasible and Π_T^* could be any random variable with support $\Omega \subseteq \mathcal{S}_{T-1}$. When the announcement is inflationary, $\varepsilon > 1$, we have $\mathcal{S}_{T-1} = [1, \varepsilon]$; when it is deflationary, $\varepsilon < 1$, we have $\mathcal{S}_{T-1} = [\varepsilon, 1]$.

The utility of household x is given by the sum of the felicity from time-t consumption and labor plus the expected continuation utility, which is evaluated for the household's worst-case scenario on the realizations of the inflation target. Formally, we assume that preferences at time t order future streams of consumption, $\mathbf{C_t} = \{c_s(h^s)\}_{s=t}^{\infty}$, and labor supply, $\mathbf{L_t} = \{l_s(h^s)\}_{s=t}^{\infty}$, so that utility is defined recursively as

$$V_t(\mathbf{C_t}, \mathbf{L_t}) = U(c_t(h^t), l_t(h^t)) + \beta_t \min_{\Omega \subseteq \mathcal{S}_t,\ G \in \mathcal{P}(\Omega)} \int_{\Omega} V_{t+1}(\mathbf{C_{t+1}}, \mathbf{L_{t+1}})\, G(d\Pi^*_{t+1}), \tag{8}$$

where $h^t = \{\Pi^*_{-\infty}, ..., \Pi^*_{t-1}, \Pi^*_t\}$ denotes history up to time t, and Ω is the support of the probability distribution G that household x ascribes to the realizations of the inflation target one period ahead, Π^*_{t+1}. Expected utility arises when the household is forced to take Ω and the associated probability distribution G as given. Under ambiguity aversion, to rank the utility from future streams of consumption and labor, the household chooses a support Ω and an associated probability distribution G so as to minimize the continuation utility V_{t+1} (worst case criterion). The support Ω is chosen among the possible realizations of the inflation target at $t+1$, denoted by $\mathcal{S}_t$. A non-degenerate set of beliefs captures the household's lack of confidence in probability assessments, with a larger set implying greater uncertainty. Given the discussion above, the probability distribution G is chosen from the set of all probability distributions $\mathcal{P}(\Omega)$ that assign positive probability to all values in the support Ω. The support of the possible realization of Π^*_T $\mathcal{S}_{T-1}$ is given by (7). There is no uncertainty about the inflation target at $t < T-1$ or at $t \geq T$. So we have $\mathcal{S}_t = 1$ $\forall t \neq T-1$. Finally, notice that since the set $\mathcal{S}_t$ is common to all households $x \in [0,1]$, they all face the same uncertainty.[4] We assume that, $\forall t$, household $x \in [0,1]$ can condition her choices to the entire history up to time t, h^t, which is fully characterized by the observed realizations of Π^*_t up to t. Household x chooses consumption plans, $c_t(h^t)$, labor supply $l_t(h^t)$ and savings $a_{t+1}(h^t)$ to maximize (8). Notice that if the realizations of the inflation target affect the consumption and labor streams of different households differently, these preferences will give rise to actions that are taken under heterogeneous beliefs. We can now define an equilibrium as follows:

Equilibrium An equilibrium is a set of beliefs, quantities, and prices such that, $\forall t$,

1. Each *household* $x \in [0,1]$ chooses c_{xt}, l_{xt}, and a_{xt+1} to maximize the utility in (8), which also determines her beliefs about the support for the next-period realizations of the inflation target, $\Omega_{xt} \subseteq \mathcal{S}_t$, and the associated probability distribution $G_{xt} \in \mathcal{P}(\mathcal{S}_t)$;

2. The *monetary authority* sets the nominal interest rate R_t as in (3);

[4] There is empirical evidence suggesting that more educated individuals and those with greater financial literacy are characterized by smaller ambiguity when investing in financial markets and dealing with financial institutions, see Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016). Here we do not allow for exogenous differences in ambiguity to better isolate the effects of wealth inequality on the formation of households' expectations, which endogenously generate heterogeneity in beliefs.

3. Each *firm* $i \in [0,1]$ sets the price $p_{it} = p_t$ optimally, after the inflation target for the period has been determined (but before any policy announcement);

4. The labor market, the goods market, and the financial market *all clear* at wage w_t, inflation Π_t, and return r_t.

3 Solution of the model

We start by assuming that the policy announcement at $t = 0$ is about the next-period inflation target Π_1^*, so that $T = 1$. We further assume that there are only two types of households differing only in initial financial wealth.[5] A fraction (half) of households are *creditors*, $j = c$, with wealth equal to $a_{x0} = a_{c0} = B > 0 \ \forall x \in [0, 1/2]$, and the remaining fraction are *debtors*, $j = d$, with financial wealth $a_{x0} = a_{d0} = -B < 0$, $\forall x \in [1/2, 1]$. Here B denotes the amount of initial *financial imbalances* in the economy. First we prove some preliminary results that clarify the functioning of the model. Then we characterize household beliefs as a function of their savings at $t = 1$. Finally, we solve for equilibrium aggregate output Y_0 and end-of-period financial imbalances B' at $t = 0$.

3.1 Preliminary results and the full credibility benchmark

Figure 1 shows the time line of the experiment. At the announcement, $t = 0$, prices are predetermined at a value normalized to one, $p_0 = 1$. The analysis focuses on characterizing output at time zero, Y_0 which is determined, given sticky prices, by the saving decisions of creditors, a_{c1}, and debtors, a_{d1}. Clearing the financial market implies that $a_{c1} = -a_{d1} = B'$, where B' denotes the amount of financial imbalances at the end of period zero. In the

Figure 1: Timing

following periods, $t \geq 1$, firm $i \in [0, 1]$ sets its price p_{it} to maximize expected profits at the

[5] Both assumptions are relaxed in the quantitative model of Section 4. To keep the notation consistent throughout the paper, we have described the economy for general T and for an arbitrary distribution of households' assets a_{xt}. In this simple model the assumption $T = 1$ entails only a minor loss of generality, because firms adjust prices in every period so output can respond just at $t = 0$. The time horizon of the announcement will matter in the quantitative model because in that case prices are adjusted slowly.

beginning of period, $d_{it} \equiv y_{it}\,(p_{it}/p_t - w_t)$, taking as given the demand for the variety of the competitive firm, which has the conventional form:

$$y_{it} = Y_t \left(\frac{p_{it}}{p_t}\right)^{-\theta}.$$

The resulting optimal nominal price is a markup over firm i's expected nominal wage:

$$p_{it} = \frac{\theta}{\theta - 1}\, E_{it}[w_t\, p_t] \quad \forall i \in [0,1], \tag{9}$$

which immediately implies $p_{it} = p_t\ \forall i$. Also, since firms set their price after observing Π^*_t, pricing decisions are taken under perfect information $\forall t \geq 1$, allowing us to conclude that

$$w_t = \frac{\theta - 1}{\theta}, \quad \forall t \geq 1. \tag{10}$$

The utility in (1), together with the preferences in (8), further implies that the labor supply of a household of type $j = c, d$ solves a simple static maximization problem, yielding the familiar condition

$$\psi_0\, l_{jt}^{\psi} = w_t. \tag{11}$$

This implies that all households (independently of wealth and beliefs) supply the same labor, which given that aggregate labor supply equals output yields $l_{jt} = Y_t,\ \forall j$. This together with (10) and (11), immediately implies that:

Lemma 1 *Output Y_t converges back to steady state at $t = 1$, so that $Y_t = \bar{Y}\ \forall t \geq 1$.*

In the Appendix we use Lemma 1 together with the interest rate rule in (3) to prove that

Lemma 2 *At any point in time $t \geq 0$, inflation is equal to the inflation target, $\Pi_t = \Pi^*_t$, and the nominal interest rate remains unchanged at its steady state value, $R_t = \bar{R}$.*

We conclude this Section by characterizing the properties of the economy in the canonical New Keynesian model in which all households fully believe the announcement. Let

$$N(Y) \equiv Y - \psi_0 \frac{Y^{1+\psi}}{1+\psi},$$

denote output net of the effort cost of working, which in equilibrium is just a monotonically increasing transformation of output Y.[6] We also denote by $\bar{N} \equiv N(\bar{Y})$ the steady state

[6]Notice that $N'(Y) > 0$ when $w < 1$, which is implied by (10).

value of $N(Y)$ and by $N_0 \equiv N(Y_0)$ the value of $N(Y)$ at time zero. We prove in the appendix that

Proposition 1 (The full credibility benchmark) *If all households fully believe the announcement, then $N_0 = \varepsilon^{\frac{1}{\sigma}} \bar{N}$. Thus output Y_0 is a strictly increasing function of ε and is independent of initial imbalances B. The new steady-state financial imbalances after implementation, B'/ε, are strictly positive and decrease relative to B if the announcement is inflationary, $\varepsilon > 1$, while they increase (relative to B) if the anouncement is deflationary, $\varepsilon < 1$.*

The proof of the proposition follows immediately by writing the Euler equation of consumption for creditors $j = c$ and debtor households $j = d$ and then imposing that the good market and the financial market should clear at the initial predetermined nominal prices.

3.2 Solving for household beliefs

Using Lemma 1 and preferences in (8), the household problem is given by

$$V(a_{j0}) = \max_{c,l,a'} \left\{ U(c,l) + \beta \min_{\Omega \subseteq \mathcal{S}_0,\ G \in \mathcal{P}(\mathcal{S}_0)} \left[\int_{\Omega} \bar{V}\left(\frac{a'}{\Pi_1^*}\right) G(d\Pi_1^*) \right] \right\} \tag{12}$$

$$\text{s.t.} \qquad c + a' \leq w_0\, l + \bar{R}\, a_{j0} + \lambda_0, \tag{13}$$

where equation (7) implies the support of possible realizations of Π_1^* is $\mathcal{S}_0 = [\min(1,\varepsilon); \max(1,\varepsilon)]$, and the continuation utility is

$$\bar{V}(a) = \frac{\left[\bar{N} + (\bar{R} - 1)\, a\right]^{1-\sigma}}{(1-\sigma)(1-\beta)}. \tag{14}$$

We notice that $\bar{V}(\cdot)$ is an increasing function of the household's wealth at the beginning of period one. Generally, higher $\Pi_1^* \in \mathcal{S}_0$ lowers continuation utility when $a' > 0$, and increases it when $a' < 0$. If $a' = 0$, households' utility is unaffected by $\Pi_1^* \in \mathcal{S}_0$. We conclude that:

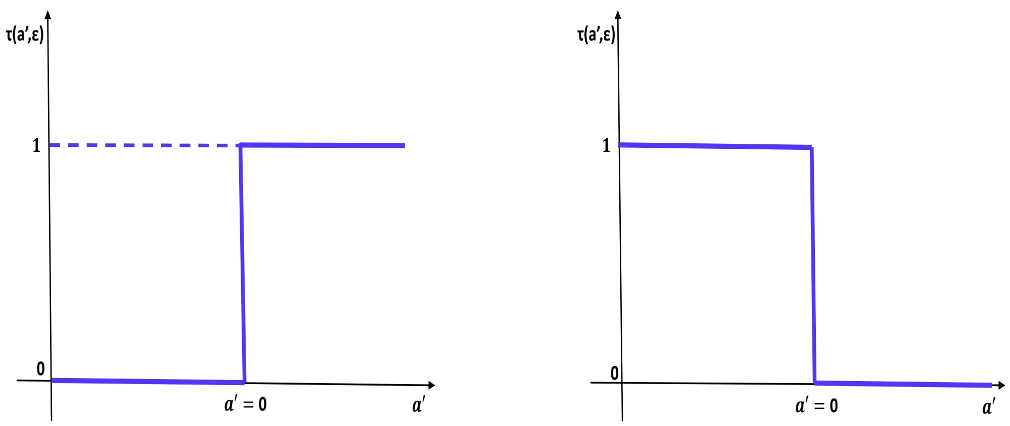
Proposition 2 (Individual beliefs) *A household-j's beliefs depend on the announcement, ε, and her end-of period savings, a'. When $a' = 0$, beliefs are indeterminate. If $a' \neq 0$, they are degenerate and equal to $\varepsilon^{\tau(a',\varepsilon)}$ where*

$$\tau(a', \varepsilon) = \mathbb{I}(\varepsilon > 1) \times \mathbb{I}(a' > 0) + \mathbb{I}(\varepsilon < 1) \times \mathbb{I}(a' < 0), \tag{15}$$

in which $\mathbb{I}$ *denotes the indicator function.*

Figure 2 characterizes the function $\tau(a', \varepsilon)$, which measures (in percentage) how much of the announcement ε the household expects to be implemented in period one. If the announcement is inflationary, $\varepsilon > 1$, $\tau(a', \varepsilon) = 1$ if $a' > 0$, and zero otherwise, which corresponds to panel (a) of Figure 2. If the announcement is deflationary, $\varepsilon < 1$, $\tau(a', \varepsilon) = 1$ if $a' < 0$, and zero otherwise, which corresponds to panel (b).

Figure 2: Endogenous determination of beliefs



(a) Case $\varepsilon > 1$ (b) Case $\varepsilon < 1$

We characterize equilibrium beliefs, output and end-of-period financial imbalances at $t = 0$. We first show that if a household starts as a creditor, $a_{x0} > 0$, then he will not be a debtor in the new steady state, $a_{x1} \geq 0$. In particular, given the definition of λ_0 and $N_0 \equiv N(Y_0)$, the equilibrium condition for labor supply in equation (11), and using the condition for optimal beliefs in equation (15), the problem of household j in (12)-(13) reduces to

$$V(a_{j0}) = \max_{a'} F(a_{j0}, a'). \tag{16}$$

where

$$F(a_{j0}, a') \equiv \frac{\left(N_0 + \bar{R}\, a_{j0} - a'\right)^{1-\sigma}}{1-\sigma} + \beta \bar{V}\left(\frac{a'}{\varepsilon^{\tau(a',\varepsilon)}}\right), \tag{17}$$

denotes household j's value for given a'. The marginal value of a household's savings, $F_2(a_{j0}, a')$, is strictly increasing in her beginning-of-period wealth a_{j0}.Intuitively, households that start as creditors have always higher incentives to save than households that start as debtor because of a consumption smoothing argument. Given this, the Appendix proves that:

Proposition 3 (No reversal in households' net financial assets) *In equilibrium, creditors and debtors never switch their net financial asset position: if $B > 0$, then $B' \geq 0$.*

We use Proposition 3 to argue that an equilibrium can be of two types: i) an equilibrium with active financial markets in the new steady state, $B' > 0$, or ii) a credit crunch equilibrium with $B' = 0$. Using Proposition 2, we can fully characterize the equilibrium beliefs in the two types of equilibria. To this aim, it is useful to define

$$\tau_c \equiv \tau(B', \varepsilon) \;\text{ and }\; \tau_d \equiv \tau(-B', \varepsilon)$$

as the equilibrium beliefs of creditors and debtors respectively. For expositional purposes, let us also define

$$\hat{\tau} \equiv (\tau_c + \tau_d)/2 \;\text{ and }\; \rho \equiv (\tau_c - \tau_d)/(2\hat{\tau}) \in [-1, 1],$$

which are related to τ_c and τ_d as follows: $\tau_c \equiv \hat{\tau}\,(1+\rho)$ and $\tau_d \equiv \hat{\tau}\,(1-\rho)$; $\hat{\tau}$ measures the *average credibility* of the announcement; while ρ measures the *correlation* between households' wealth and their perception of the announcement's credibility. When $\rho > 0$, creditors believe the announcement more than debtors; and conversely when $\rho < 0$; $\rho = 0$ means that all households share the same beliefs. We have:

Proposition 4 (Equilibrium beliefs) *In a credit crunch equilibrium, $B' = 0$, households' beliefs are indeterminate. In any other equilibrium, $B' > 0$, only one type of household believes the announcement, $\hat{\tau} = 1/2$: if the announcement is inflationary, creditors believe it, $\rho = 1$; if it is deflationary, debtors believe it, $\rho = -1$. So in general we have*

$$\rho = \rho(\varepsilon) \equiv 1 - 2\mathbb{I}(\varepsilon < 1). \tag{18}$$

We next provide conditions on the exogenous parameters of the model to determine which of the two types of equilibrium can be sustained. A necessary condition for an equilibrium with active credit markets, $B' > 0$, is that the intertemporal Euler equation determining the solution to the household problem in (16)-(17) is satisfied with an equality for both creditor and debtor households. After maximizing (16) we obtain the following two conditions:

$$\frac{\bar{N} + (\bar{R}-1)\varepsilon^{-\hat{\tau}\,(1+\rho)}\,B'}{N_0 + \bar{R}B - B'} = \varepsilon^{-\hat{\tau}\,\frac{1+\rho}{\sigma}}\;, \tag{DA}$$

$$\frac{\bar{N} - (\bar{R}-1)\varepsilon^{-\hat{\tau}\,(1-\rho)}\,B'}{N_0 - \bar{R}B + B'} = \varepsilon^{-\hat{\tau}\,\frac{1-\rho}{\sigma}}\;. \tag{SA}$$

Equation (DA) can be interpreted as creditors' demand for assets: the demand for assets B' is increasing in time-zero net output N_0, because creditors want to save more when output increases. By the same logic, equation (SA) characterizes the supply of assets by debtors: the supply of assets B' is decreasing in N_0, as debtors want to borrow less (save more) when time-zero output is higher. The intersection of equations (DA) and (SA) imply N_0 is given by the function $N_0(\varepsilon, \hat{\tau}, \rho)$,

$$N_0 = N_0(\varepsilon, \hat{\tau}, \rho) \equiv \bar{N} \left[\omega \, \tilde{\varepsilon}^{\,\frac{1+\rho}{\sigma}} + (1-\omega)\, \tilde{\varepsilon}^{\,\frac{1-\rho}{\sigma}}\right] + B\,\zeta \left[\tilde{\varepsilon}^{\,(1+\rho)\left(\frac{1}{\sigma}-1\right)} - \tilde{\varepsilon}^{\,(1-\rho)\left(\frac{1}{\sigma}-1\right)}\right], \quad (19)$$

where $\tilde{\varepsilon} \equiv \varepsilon^{\hat{\tau}}$ measures the announcement rescaled by its average credibility with

$$\omega \equiv \frac{1 + (\bar{R}-1)\, \tilde{\varepsilon}^{\,(1-\rho)\left(\frac{1}{\sigma}-1\right)}}{2 + (\bar{R}-1)\left[\tilde{\varepsilon}^{\,(1+\rho)\left(\frac{1}{\sigma}-1\right)} + \tilde{\varepsilon}^{\,(1-\rho)\left(\frac{1}{\sigma}-1\right)}\right]} \in [0,1],$$

$$\zeta \equiv \frac{\bar{R}\,(\bar{R}-1)}{2 + (\bar{R}-1)\left[\tilde{\varepsilon}^{\,(1+\rho)\left(\frac{1}{\sigma}-1\right)} + \tilde{\varepsilon}^{\,(1-\rho)\left(\frac{1}{\sigma}-1\right)}\right]} > 0.$$

The necessary and sufficient condition for equation (SA) and equation (DA) to cross at $B' > 0$ is that

$$B > \frac{|\varepsilon^{\frac{1}{\sigma}} - 1|\bar{N}}{2\bar{R}}. \quad (20)$$

The latter obtains when the intercept of the (SA) equation , evaluated at the equilibrium beliefs as given by (18),

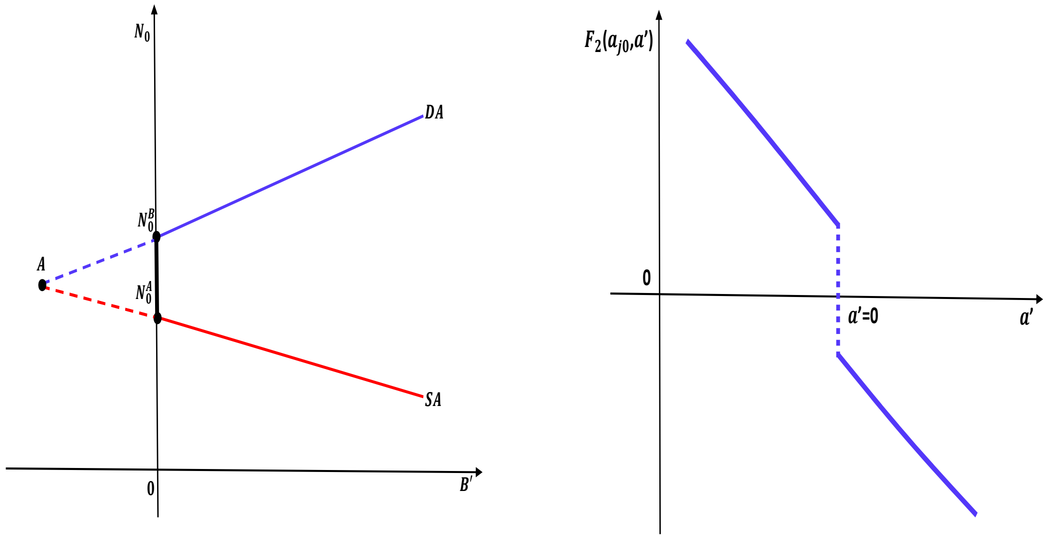
$$N_0^A \equiv \min\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} + \bar{R}B, \quad (21)$$

is above the intercept of the (DA) equation,

$$N_0^B = \max\{1, \varepsilon^{\frac{1}{\sigma}}\}\bar{N} - \bar{R}B. \quad (22)$$

But when (20) fails, we have $N_0^A < N_0^B$, which implies that (SA) and (DA) would intersect at a point where $B' < 0$, as in panel (a) of Figure 3. In this case, given Proposition 3, we have a credit crunch equilibrium, $B' = 0$, where both households types $j = c, d$ completely undo their financial positions. This equilibrium arises because of the endogenous beliefs of household j (see Proposition 2), which cause a discontinuous fall in the expected return on assets when the household j's savings switch from negative to positive. So at $a' = 0$ the value of household j's savings $F(a_{j0}, a')$ has a kink and the marginal value of savings, $F_2(a_{j0}, a')$, falls discontinuously. In a credit crunch equilibrium, $F_2(a_{j0}, a')$, changes sign at $a' = 0$, as in panel (b) of Figure 3, which guarantees that household j will find it optimal neither to borrow—which means higher consumption today in exchange for lower

Figure 3: Credit crunch equilibrium



(a) Financial market

(b) Household-j's maximization

consumption tomorrow— nor to lend—lower consumption today and higher tomorrow. On these premises, in the Appendix we prove that:

Lemma 3 *If* (20) *fails, then* $N_0^A < N_0^B$, *and the equilibrium features a credit crunch* $B' = 0$.

Intuitively, credit crunches arise because of a *zen effect*, due entirely to the endogenous formation of households' beliefs under ambiguity aversion: due to the kink in the value of their savings $F(a_{j0}, a')$, households naturally tend to choose a financial position that assures them "complete peace of mind" about future monetary policy choices, which in this simple model is attained when $a' = 0$.

Lemma 3 together with the foregoing considerations immediately implies:

Proposition 5 (Equilibrium output) *An equilibrium always exists. If* (20) *holds, then the financial market is active,* $B' > 0$, *and net output,* $N_0 = N(Y_0)$, *is given by the point where the demand of assets* (DA) *and the supply of assets* (SA) *intersect, evaluated at the equilibrium beliefs of Proposition 4, so that* $N_0 = N_0(\varepsilon, 1/2, \rho(\varepsilon))$. *If* (20) *fails, households' beliefs are indeterminate and the equilibrium features a credit crunch,* $B' = 0$, *where net output,* N_0, *can be any value in the range* $[N_0^A, N_0^B]$, N_0^A *and* N_0^B *being given by* (21) *and* (22), *respectively.*

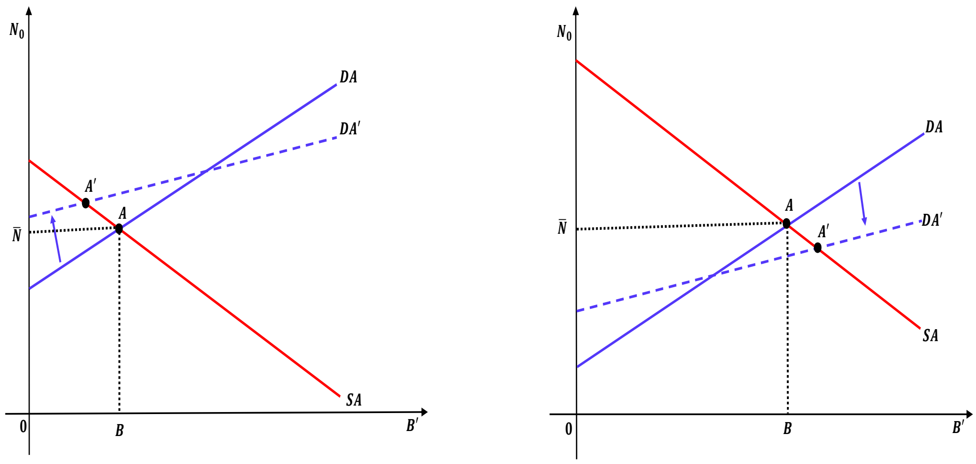
3.3 The effect of ambiguity on monetary policy announcements

We next characterize equilibrium output in our model with ambiguity averse households. We focus first on the case where condition (20) holds and the equilibrium is such that $B' > 0$ so that $N_0 = N_0(\varepsilon, 1/2, \rho(\varepsilon))$. The first term on the right-hand side of (19) is always positive and characterizes the intertemporal substitution effect on consumption. The second term characterizes the effects on consumption of redistributing expected future wealth from one household type to the other. This second term is zero when $B = 0$ or $\sigma = 1$, because no wealth is redistributed or wealth and substitution effect exactly offset each other. So when $B = 0$ or $\sigma = 1$, net output is $N_0 = \varepsilon^{\frac{\hat{\tau}}{\sigma}} \bar{N}$, which is as in the full credibility benchmark (see Proposition 1) but for the scaling effect of $\hat{\tau} = 1/2$ on the elasticity of net output to the policy announcement. The *scaling effect* of ambiguity on the transmission of monetary policy is easy to interpret: when for instance $\varepsilon > 1$, only a half of households trust the announcement of lower real interest rates and therefore increases demand because of the substitution between savings and consumption.

When $B > 0$ and $\sigma > 1$, the second term is strictly negative in response to both an expansionary announcement, $\varepsilon > 1$, and an announcement of monetary tightening, $\varepsilon < 1$, when evaluated at the equilibrium beliefs of Proposition 4. In particular, we have $\rho = 1$ when $\varepsilon > 1$, and $\rho = -1$ when $\varepsilon < 1$. This comovement between ρ and ε implies that, on top of to the scaling effect, there is also a *selection* effect. It's not a random half of the population that trusts the announcement, but it's the half that looses the most or benefit the least due to the redistributive effects of monetary policy. When B and/or σ are high enough the *selection* effect becomes more important, to the point that an announcement of an expansionary monetary policy $\varepsilon > 1$ can even cause a fall in aggregate demand at $t = 0$. This gives rise to a *misguidance* effect which is measured by the opposite of the second term in (19).

We now use the diagram representation in Figure 4 to clarify how the initial financial imbalances, B, and the correlation between a household's wealth and its beliefs, ρ, affect output at time zero in the case of an inflationary announcement, $\varepsilon > 1$. In this case, the supply of assets (SA) remains unchanged because debtors act on the belief that the announcement will not be implemented ($\Pi_1^* = 1$), while the demand (DA) can shift up or down: if B is small, the substitution effect prevails and (DA) shifts up (at least locally), as in panel (a) of Figure 4; if B is large, the income effect prevails and (DA) shifts down (locally), as in panel (b). Case (b) arises because creditors, expecting a lower return on assets, feel poorer and consume less, leading to a contraction in current aggregate net income. This, in turn, induces debtors to borrow more to smooth consumption, which increases their supply of assets and allows the financial market to clear, even if debtors'

Figure 4: Clearing of financial markets after an inflationary announcement



(a) $\varepsilon > 1$ and low B (b) $\varepsilon > 1$ and high B

Notes: The left (right) panel depicts the supply and demand of assets before and after an expansionary announcement $\varepsilon > 1$ at $t = 0$, with low (high) level of financial imbalances B The solid blue and red lines represent, respectively, the demand and supply of assets before the announcement. The dashed blue line represents the demand of assets after the announcement.

expected cost of debt service does not change. This corresponds to point A' in panel (b).

We notice that for debtors the income and the substitution effects both work in the same direction, so the shift in asset supply (SA) is unambiguously signed. For example we can see that, when $\rho < 0$, a deflationary announcement $\varepsilon < 1$ is always contractionary in output: under $\varepsilon < 1$ and $\rho < 0$, the right-hand side of (19) is linear in B, with an intercept lower than $\bar{N}$ and a negative slope. Combining the results of Propositions 6 and 1, the next corollary summarizes this discussion:

Proposition 6 (Equilibrium output) *After an inflationary announcement $\varepsilon > 1$, output Y_0 increases less than in the full credibility benchmark. This difference is increasing in B, and Y_0 can even decrease if B is large enough. In response to a deflationary announcement, $\varepsilon < 1$, Y_0 always decreases. The decrease is larger the larger is B; and if B is large enough, Y_0 decreases more than in the full credibility benchmark.*

We notice that in the case of an inflationary announcement ambiguity unambiguously reduces the effect of the policy announcement on output, as it dampens the positive effect of the announcement through the substitution channel due to the fact that only half of the population trust it, and introduces a negative wealth effect due to the fact that only creditors believe the announcement. In the case of a deflationary announcement these two effects work in opposite directions. The wealth effect amplifies the negative impact on output as debtors would be the one trusting the higher real interest rates, while the dampening of the substitution effect reduces the negative effect of the announcement on output.

Finally, we briefly comment on how ambiguity on the credibility of the policy announcement affects the transmission to output when $B > 0$ but condition (20) fails so that we have a credit crunch. In a credit crunch equilibrium caused by an inflationary announcement $\varepsilon > 1$, output never falls, as $N_0 \geq N_0^A > \bar{N}$, but it is always lower than the output level of the full credibility benchmark, as the failure of (20) implies $N_0^A < N_0^B < \varepsilon^{\frac{1}{\sigma}}\bar{N}$. In a credit crunch equilibrium caused by a deflationary announcement $\varepsilon < 1$, output always falls, as $N_0 < N_0^B < \bar{N}$, but it is always higher than the output of the full credibility benchmark.

In the Appendix, we compare the steady state imbalances that result when the announcement is implemented, B'/ε, with the corresponding imbalances in the canonical New Keynesian model, where the announcement is fully credited by all households.

3.4 Discussion

We now briefly discuss some properties and extensions of the analytical model, with the theoretical details reported in the Appendix. Section 6 studies the quantitative implications of some of these extensions.

Long vs short term nominal bonds We assumed that households can save or borrow just in a one period bond that pays a pre-specified nominal interest rate (short term nominal bonds). But as shown in Lemma 2 the nominal interest rate remains unchanged over time. This means that households disagree just on future expected inflation (not on future short term nominal interest rates), so allowing households to trade in nominal bonds at different maturities would have no equilibrium effects.

Real asset In the Appendix we study a model where households can also trade in a real asset which is in fixed supply (say a Lucas tree) and pays with certainty a per period return equal to $\beta^{-1} - 1$. Households face some convex costs in adjusting their holdings of the real asset. When households disagree on the expected real return of financial assets, trading in the real asset is profitable. After a monetary announcement (either inflationary $\varepsilon > 1$ or deflationary $\varepsilon < 1$), the expected real interest rate on financial assets is generally lower for creditors than for debtors (see Proposition 2), so the real asset tends to be reallocated from debtors to creditors. Compared with the baseline model, after an announcement, output is higher and a credit crunch equilibrium with $B' = 0$—which tends to arise when adjustments costs are small enough—is more likely. But the misguidance effect is still present: output is decreasing in the initial financial imbalances B and, after an inflationary announcement, it is always smaller than in the full credibility benchmark.

Government bonds We assumed that the supply of bonds is entirely determined by households. In practice households also also hold government bonds in their portfolio and at least since Ricardo (1888) and Barro (1974) there is genuine doubt whether households perceive government bonds as net wealth. If households feel liable for their country's public debt, households are poorer and an inflationary announcement is more likely to be expansionary. So a failure of the Barro-Ricardo equivalence proposition has important implications for the functioning of monetary policy in our model.

Policy implications When debtors and creditors share the same beliefs, they also have the same marginal propensity to consume and redistributing wealth has no effects on aggregate consumption. But in response to an inflationary announcement, their beliefs are different and taxing today the creditors to transfer the resulting income to the debtors is expansionary. Intuitively redistributive policies are equivalent to reducing the level of initial imbalances B, and with enough redistribution forward guidance can become as expansionary as in the full credibility benchmark. Generally forward guidance is more expansionary when accompanied by redistributive policies.

Liquidity traps For expositional simplicity we assumed that the economy is initially in a steady state equilibrium. The analysis would go through almost unchanged if considering an economy which is initially in a liquidity trap, say because at time zero the household discount factor β_0 is so high that the nominal interest rate in (3) is at the zero lower bound—while the economy is back to steady state in period one with $\beta_t = \beta < 1$, $\forall t \geq 1$.

Modelling of ambiguity aversion Households have Maximin preferences as in Gilboa and Schmeidler (1989) but, after an inflationary announcement, the expected inflation of creditors would respond more than the expected inflation of debtors also under alternative models of ambiguity aversion, including the multiplier preferences proposed by Hansen and Sargent (2001, 2008), whose axiomatic foundations are provided by Strzalecki (2011).

Different source of uncertainty We assumed that households doubt about the credibility of the monetary authority γ. When households face uncertainty about future inflation—say about the inflation target θ that the monetary authority knows it will set in period T in the absence of the announcement—an inflationary announcement $\varepsilon > 1$ is always expansionary. To see this assume that γ is known and that households think that the central bank sets Π_T^* to minimize

$$\boldsymbol{L}_1 = \left(\hat{\Pi} - \Pi_T^*\right)^2 + \gamma \left(\epsilon - \Pi_T^*\right)^2, \tag{23}$$

where ϵ is the monetary announcement and $\hat{\Pi} \in [\hat{\Pi}^l, \hat{\Pi}^u]$ is the inflation target about which households face Knightian uncertainty. Given (23) households infer that

$$\Pi_T^* = \frac{\hat{\Pi} + \gamma\varepsilon}{1+\gamma}$$

which implies that the support of the feasible values of Π_T^* is equal to

$$\mathcal{S}_{T-1} = \left[\frac{\hat{\Pi}^l + \gamma\varepsilon}{1+\gamma}, \frac{\hat{\Pi}^u + \gamma\varepsilon}{1+\gamma}\right]. \tag{24}$$

This means that an inflationary announcement $\varepsilon > 1$ increases the set of feasible inflation rates for all households in the economy independently of their initial financial position, which implies a fall in expected real rates for all households. Because of this the announcement is always expansionary relative to the status quo.

In practice households face uncertainty about both future inflation $\hat{\Pi}$ and the credibility of the announcement γ. The relative response of the inflation expectations of creditors and debtors identifies the empirically relevant source of uncertainty. If uncertainty is mostly about the credibility of the monetary authority γ, as in (7), after an inflationary announcement, the inflation expectations of creditors increase more than the inflation expectations of debtors. When instead uncertainty is mostly about future inflation, as in (24), the disagreement in inflation expectations between creditors and debtors falls after the announcement. The empirical evidence reported below indicates that, at the time of the start of forward guidance by the ECB, there was substantial uncertainty about the credibility of the announcement, which is coherent with the large disagreement among European households in how much the ECB could be trusted, see for example Guiso, Sapienza, and Zingales (2016).

Bounds on credibility We assumed that households regard as possible any γ between zero (no credibility) and infinity (full credibility). When credibility is bounded to be in the interval $[\gamma^l, \gamma^u]$, the set of inflation targets Π_T^* that households regard as feasible after an inflationary announcement $\varepsilon > 1$ is given by

$$\mathcal{S}_{T-1} = \left[\frac{1 + \gamma^l\varepsilon}{1+\gamma^l}, \frac{1 + \gamma^u\varepsilon}{1+\gamma^u}\right]. \tag{25}$$

In practice the lower bound of (25) determines the increase in expected inflation common to all households, while the size of the interval measures the uncertainty following the announcement. After forward guidance, the interval in (25) can be identified by using evidence on the changes in expected inflation and in the correlation between expected inflation and net financial assets. Knowing (25) is enough to characterize the effects of the announcement in the model, but notice that, without any additional assumptions, the bounds on credibility, γ^l and γ^u, and the announcement ε are not separately identified. In the quantitative analysis below, we will be assuming that households believe that full credibility is possible $\gamma^u = \infty$, which implies a conservative estimate for the size of the inflationary announcement ε.

4 The quantitative model

We perform a quantitative analysis of the relevance of our mechanism by studying the effects on the economy of a forward guidance announcement on the path of the monetary policy instrument, namely the short-term nominal interest rates. For this purpose we extend the model in several directions. In particular, we allow for (i) household to trade both in nominal bonds and real assets, (ii) a generic initial household distribution of assets, (iii) sticky prices à la Rotemberg (1982), and (iv) a liquidity trap. Extensions (i) allows to capture the different exposure to interest rate and inflation risk of different portfolio combinations of financial and real assets; (ii) to match the observed distribution of households' assets; (iii) to obtain a conventional new-Keynesian Phillips curve; and (iv) to characterize the state of the European economy at the time of the announcement. We next describe the economy, then characterize the equilibrium in the liquidity trap before the announcement and finally turn to the response of the economy after the announcement.

Real and financial assets There is a fixed supply H of a *real* asset which, in every period, pays a per unit return $\nu > 0$ with certainty. Households, still indexed by $x \in [0, 1]$, pay a convex cost for adjusting their holdings of the real asset

$$\chi\left(h_{xt+1}, h_{xt}\right) = \frac{\chi_0}{2}\left(\Delta h_{xt}\right)^2 h_{xt}, \tag{26}$$

where $\Delta h_{xt} \equiv (h_{xt+1} - h_{xt})/h_{xt}$ is the percentage change in the real asset holdings of household x. Clearing in the market for the real asset implies that

$$\int_0^1 h_{xt}\, dx = H. \tag{27}$$

Household-x can trade in a one period nominal bond that pays an interest rate r_t in period t, so $\forall t$ the household-x budget constraint is given by

$$c_{xt} + a_{xt+1} + q_t\, h_{xt+1} + \chi(h_{xt+1}, h_{xt}) \leq w_t\, l_{xt} + (q_t + \nu)\, h_{xt} + r_t\, a_{xt} + s_t, \tag{28}$$

where q_t is the price of the real asset at time t, and s_t is a transfer from the government. The financial market is characterized by a mutual fund that collects all interest payments by borrowers and all firm profits and pays interests to owners of the asset. The fund also pays an amount Υ to external agents, who represent foreign holders of domestic assets. We use the parameter Υ to match the holdings of financial assets by European households in the data. The fund is owned by the government which rebates aggregate profits (or losses) S_t to households, with $S_t = 0$ in steady state. There are no frictions in trading the financial asset, and lenders and borrowers are subject to the same interest rate. The flow budget constraint of the fund equates aggregate net interest payments, $(r_t - 1) \int_0^1 a_{xt}\, dx + \Upsilon$, plus profits of the fund to the sum of dividends, D_t, and the net new supply of assets, $\int_0^1 (a_{xt+1} - a_{xt})\ dx$ so that

$$r_t \int_0^1 a_{xt}\, dx + \Upsilon + S_t = \int_0^1 a_{xt+1}\, dx. \tag{29}$$

Sticky prices Each firm $i \in [0, 1]$ can adjust nominal prices subject to convex adjustment costs, as in Rotemberg (1982). Adjustment costs are quadratic in the rate of price change and are scaled by aggregate output, Y_t:

$$\kappa\left(\pi_{it}, Y_t\right) = \frac{\kappa_0}{2}\left(\pi_{it}\right)^2 Y_t, \tag{30}$$

where $\pi_{it} = (p_{it} - p_{it-1})/p_{it-1}$ denotes the inflation rate for firm i and $\kappa_0 > 0$.

Liquidity trap To model the liquidity trap, we follow among others Eggertsson and Woodford (2003) and Christiano, Eichenbaum, and Rebelo (2011) in assuming that the nominal interest rate in (3) is at the zero lower bound at $t = 0$ due to a temporary (unforeseen) increase in the households subjective discount factor β_t. In particular we assume that the economy is in steady state at $t = 0$ and that β_t evolves as follows:

$$\beta_t = \begin{cases} \hat{\beta} & \text{if } \ t \in [0, t_\beta] \\ \beta & \text{otherwise} \end{cases} \tag{31}$$

with $\hat{\beta} > 1 > \beta$ and $R_t = 1\ \forall t \in [0, t_\beta]$.

4.1 The economy in a liquidity trap

We characterize the equilibrium of the economy at $t \geq 0$, after the shock to β_t has been realized, under the assumption that monetary policy follows the interest rate rule in (3) with $\Pi_t^* = 1$, implying that $R_t = \hat{R}_t$, $\forall t$, where

$$\hat{R}_t \equiv \min\left(1, \frac{1}{\beta_t}\,{\Pi_t}^{\phi}\right). \tag{32}$$

The household problem Given perfect foresight, household-x chooses consumption, labor supply, financial and real asset holdings at $t = 0$ to maximize

$$V(a_{xt}, h_{xt}; \hat{R}_t) = \max_{\{c_{xs}, l_{xs}, a_{xs+1}, h_{xs+1}\}_{s \geq t}} \sum_{s=0}^{\infty} \beta_s\, U(c_{xs}, l_{xs}), \tag{33}$$

subject to the flow budget constraint in (28) and to the non-negativity constraint on real assets, $h_{xt} \geq 0$. Maximizing with respect to l_{xt} yields (11), while maximization with respect to h_{xt+1} yields

$$\frac{q_{t+1} + \nu + \chi_0\, \Delta h_{xt+1}}{q_t + \chi_0\, \Delta h_{xt}} = r_{t+1}. \tag{34}$$

Equation (34) requires that, under perfect foresight, the financial asset and the real asset pay the same return. The perfect foresight together with the adjustment cost, imply that no trade in real assets is optimal, $\Delta h_{xt} = 0$, at the sequence of prices $q_{t+1} + \nu = q_t\, r_{t+1}$. The first order condition for a_{xt+1},

$$\left(c_{xt} - \psi_0 \frac{l_{xt}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta\, r_{t+1} \left(c_{xt+1} - \psi_0 \frac{l_{xt+1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{35}$$

together with equation (11) and the intertemporal budget constraint of household-x, allows to solve for c_{xt}. Aggregating the consumption choices of all households x, we obtain that aggregate consumption, $C_t \equiv \int_0^1 c_{xt} dx$, is equal to

$$C_t = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \left(\sum_{s=t}^{\infty} b_{ts}^{\frac{1}{\sigma}}\, m_{ts}^{1-\frac{1}{\sigma}}\right)^{-1} \sum_{s=t}^{\infty} m_{ts} \left(\frac{\psi_0 \psi}{1+\psi} Y_s^{1+\psi} + D_s - \Upsilon + \nu\, H\right) \tag{36}$$

where $m_{ts} \equiv \left(\prod_{u=t+1}^{s} r_u\right)^{-1}$ is the price at t of one unit of output paid in period s, while $b_{ts} \equiv \prod_{u=t+1}^{s} \beta_u$ is the discount factor over the time period $s - t$.

The firm problem In each period t, firm-i sets its nominal price p_{it} to maximize the discounted sum of present and future profits, with the cost of price adjustment depending on the previous period price:

$$W(p_{it-1};\hat{R}_t) = \max_{\{p_{is}\}_{s\geq t}} E_f\left\{\sum_{s=t}^{\infty} m_{ts}\,Y_t\,\left[\left(\frac{p_{it}}{p_t}-w_t\right)\left(\frac{p_{it}}{p_t}\right)^{-\theta}-\frac{\kappa_0}{2}\left(\frac{p_{it}}{p_{it-1}}-1\right)^2\right]\right\}, \quad (37)$$

where $E_f[\cdot]$ is the expectation operator conditional on firms' beliefs.[7] The solution to the firm's problem implies symmetric pricing, $p_{it} = p_t\ \forall i,t$, which can be used to derive the following standard new-Keynesian Phillips curve:

$$1-\kappa_0\,(\Pi_t-1)\,\Pi_t+\kappa_0\,E_f\left[m_{t,t+1}\,(\Pi_{t+1}-1)\,\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]=\theta\,(1-w_t). \quad (38)$$

Using symmetry, the aggregate profits are equal to

$$D_t=\left[1-w_t-\frac{\kappa_0}{2}\,(\Pi_t-1)^2\right]\,Y_t. \quad (39)$$

Equilibrium Clearing of the goods market implies that

$$Y_t+\nu\,H=C_t+\frac{\kappa_0}{2}\,\pi_t^2\,Y_t+\frac{\chi_0}{2}\int_0^1\,(\Delta h_{xt})^2\;h_{xt}\,dx+\Upsilon. \quad (40)$$

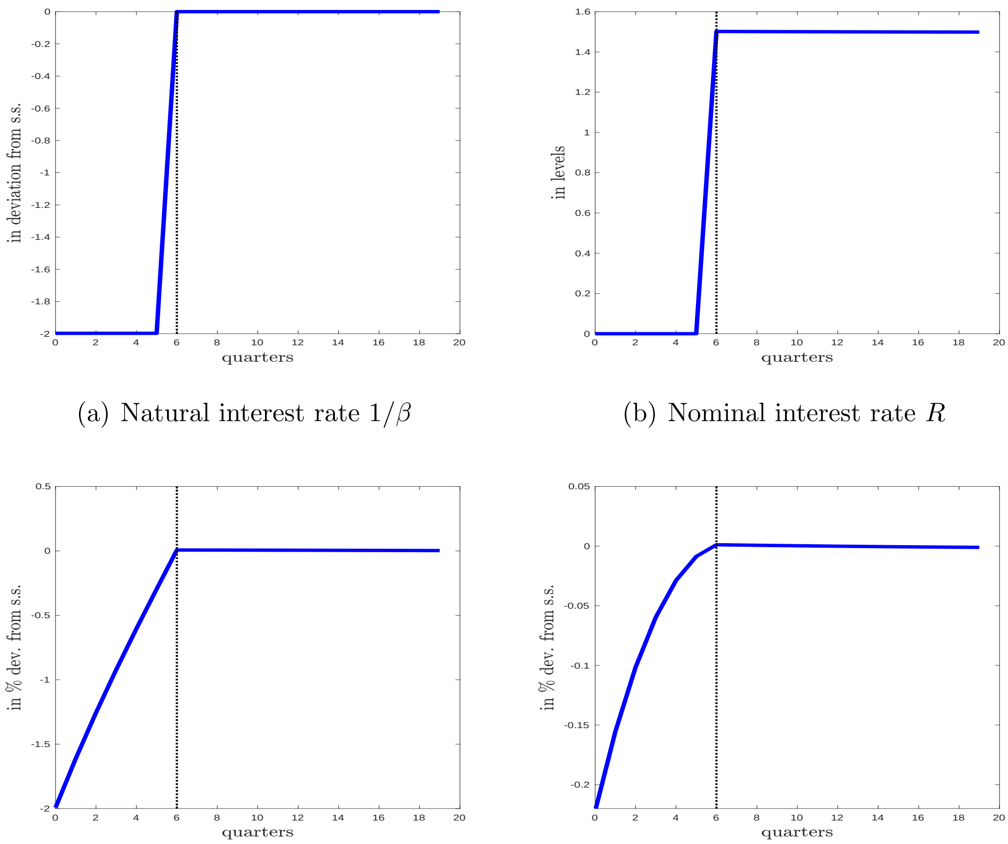
In general we have characterized the equilibrium of the economy in a liquidity trap as follows:

Lemma 4 (Equilibrium under homogeneous beliefs) *When β_t evolves as in* (31), *the economy is fully characterized by the tuple $[D_t, Y_t, w_t, C_t, \Pi_t, R_t, r_t, q_t]$, where (i) D_t and Y_t are given by* (39) *and* (40)*; (ii) aggregate labor supply and consumption solve a representative household problem that yields* (11) *and* (36)*; (iii) inflation Π_t satisfies the Phillips curve in* (38) *under perfect foresight; (iv) the nominal interest rate is given by* (32)*; (v) the real interest rate satisfies the identity $r_t = R_{t-1}/\Pi_t$; and (vi) there is no trade in real assets, implying that $q_t = (q_{t+1}+\nu)/r_{t+1}$ satisfies both* (27) *and* (34).

Figure 5 characterizes key properties of the economy. The shock to the subjective discount factor β_t causes a recession and a deflation over the time interval $[0, t_\beta]$. As soon as β_t stabilizes back to its steady state value to β, at $t = \hat{T}+1$, the economy goes back to steady state. The shock to the discount factor β_t affects the household distribution of real and financial assets, but the distribution has no effect on aggregate output, consumption and inflation. We state these results formally in the following Proposition:

[7] This is redundant notation under perfect foresight, but it will be useful when we will allow for uncertainty.

Figure 5: Impulse responses to the discount factor shock



(a) Natural interest rate $1/\beta$ (b) Nominal interest rate R

(c) Output Y (d) Inflation π

Notes: Impulse responses to an unforeseen shock to β_t. All responses are expressed in quarterly % deviation from the steady state value. The vertical dashed lines denote $t = t_\beta + 1$. The economy is calibrated as described in Table 3.

Proposition 7 (Liquidity trap) *When β_t evolves as in (31) and agents have perfect foresight about the path of $R_t = \hat{R}_t$, output is unaffected by the household distribution of real and financial assets, and the economy is back to steady state at $t = t_\beta + 1$. The equilibrium nominal interest rate and output are such that $\forall t = 0, 1, ...t_\beta$ $R_t = 1$, $\Pi_t < 1$ and $Y_t < \bar{Y}$ while $R_t = 1/\bar{\beta}$, $\Pi_t = 1$ and $Y_t = \bar{Y}$ $\forall t > t_\beta$.*

4.2 The economy after the forward guidance announcement

The announcement is made in period $t = 0$ after the shock to the path of the discount factor β_t has been realized. In absence of any monetary policy announcement, the equilibrium interest rate given by the policy rule $\hat{R}_t$ in equation (32) would be $R_t = 1$ for $t \leq t_\beta$ and $R_t = \bar{R}$ for $t > t_\beta$. This is because the shock to the discount factor β_t makes the lower bound on nominal interest rate binding at any $t \leq t_\beta$, so that the economy exits the liquidity trap at $t = t_\beta + 1$. The forward guidance announcement we study is such that, once out of the liquidity trap, the monetary authority promises it will keep the nominal interest rate lower than it would have otherwise done given its *normal-times* policy, i.e. $R_t = R^* < \bar{R}$, until $t = t_r > t_\beta$. We notice that, as a result of the announcement, the economy may exit the liquidity trap before $t = t_\beta + 1$. In particular, let $T \leq t_\beta + 1$ denote the first date at which, given the announcement, $\hat{R}_T > 0$ is implied by equation (32); T is an endogenous variable. This means that the announced path of the nominal interest rate, R_t^a, is as follows

$$R_t^a = \begin{cases} R^* & \text{if } \; t \in [T, t_r] \\ \hat{R}_t & \text{otherwise} \end{cases}. \tag{41}$$

Combining R_t^a with the equilibrium values of $\hat{R}_t$ at $t < T$ and $t > t_r$ that follow from Proposition 7, we obtain that the announced path of the nominal interest rate is a monotonically increasing step function such that $R_t^a = 1$ for $t \in [0, T-1]$, $R_t^a = R^*$ for $t \in [T, t_r]$, and $R_t = \bar{R}$ otherwise.

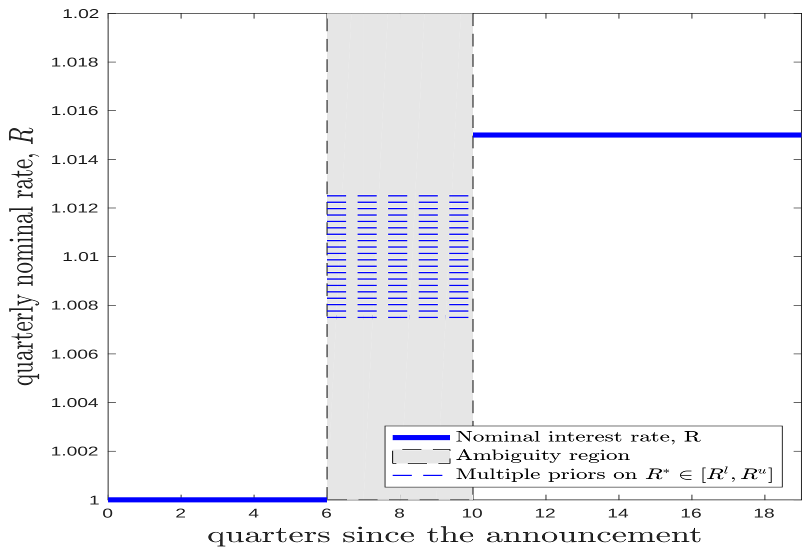
Ambiguity Households and firms doubt about the credibility of the announcement and about whether the monetary authority will deviate from her normal interest rate rule in (32). As a result, households face uncertainty about the value of the nominal interest R^* that the monetary authority will set over the period $[T, t_r]$ and have multiple priors about it. As discussed above, we assume that full credibility is possible but also that there is a lower bound on the credibility of the monetary authority, so R^* could be any random variable with support $\Omega \equiv [R^l, R^u]$. The larger the interval $R^u - R^l$, the greater the amount of uncertainty. Figure 6 illustrates the possible paths of R_t, for a given set Ω.

The household problem The household problem at $t \geq T$ is the same as described in equation (33) because ambiguity is resolved by then, and she has perfect foresight on the path of economic variables. At $t < T$, instead, households face uncertainty and potentially disagree about the realization of economic variables at $t \geq T$. We notice households face no uncertainty and agree about the realization of equilibrium prices at $t < T$. Thus we can characterize the household-x at $t = 0$ as,

$$V_0\left(a_{x0}, h_{x0}\right) = \max_{\{c_{xt}, l_{xt}, a_{xt+1}, h_{xt+1}\}_{t=0}^{T-1}} \left\{ \sum_{t=0}^{T-1} \hat{\beta}^t \, U(c_{xt}, l_{xt}) \; + \hat{\beta}^T \min_{G \in \mathcal{P}(\Omega)} \int_\Omega V\left(a_{xT}, h_{xT}, R^*\right) G(dR^*) \right\},$$

where $\Omega = [R^l, R^u]$ is the support of the prior probability distributions of R^*, subject to the flow budget constraint in equation (28), and to $h_{xt} \geq 0$ and $a_{xt} \geq \underline{a}$ for all t. The household

Figure 6: Multiple priors on R_t

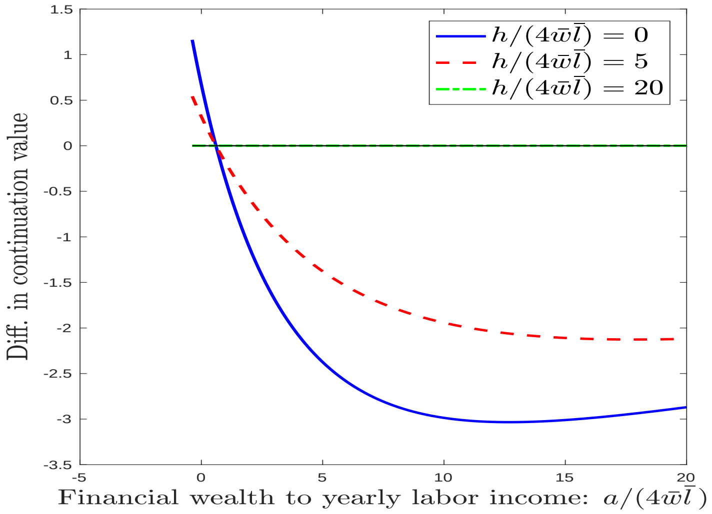


Notes: Households face (Knightian) uncertainty about the evolution of nominal interest rates in the time period depicted in grey. The solid blue line plots the path of R_t outside of the period of uncertainty, $t < T$ and $t > t_r$. In this example $T = 6$ and $t_r = 10$. The dashed blues lines represent the possible paths of $R_t = R^*$ in the interval $R^* \in [R^l, R^u]$ over which that agents have multiple priors.

internalizes that her choice will affect her worst-case beliefs about the implementation of the policy announcement at $t = T$. In particular, $V(a, h, R^*)$ is the household's continuation value when uncertainty is resolved at $t = T$, as defined in equation (33). The exposure of $V(a, h, R^*)$ to the realization of R^* depends on the combination of (a, h) that the household chooses to have at $t = T$. As a result, the worst-case beliefs are a function of the chosen combination of (a_{xT}, h_{xT}).

To illustrate the effect of policy on the households' continuation values, we plot in Figure (7) the difference between $V(a, h; R^*)$ evaluated at $R^* = R^l$ and $V(a, h; R^*)$ evaluated at $R^* = R^u$ as a function of the same values of a and h. In the plot we use the parameter values of Table 3 discussed later. We consider three values of h, equal to zero, one and five times yearly labor income of steady state, respectively. We consider a grid of values for the financial asset a, between zero and 20 times yearly labor income. The figure shows that the difference between $V(a, h; R^l)$ and $V(a, h; R^u)$ is decreasing in a for given h, so that there exists a threshold function $a^*(h)$ such that $V(a, h; R^u) < V(a, h; R^l)$ if and only if $a > a^*(h)$. So, given the worst case criterion, only sufficiently wealthy households believe that the monetary authority will implement the announcement. We emphasize that the threshold $a^*(h)$ is strictly positive, about two times yearly labor income. This is because an expansionary monetary policy, if implemented, increases labor income, which is beneficial to the household. Financial wealth has to be high enough that the loss from the lower capital income associated to lower real rates dominates the gains from higher labor income, for the household to be a net loser. Finally, we notice that the threshold $a^*(h)$ is increasing in h, but quantitatively very little. For a given leve of financial asset, higher real assets make the implementation of monetary policy relatively less important for the household payoff, as the the real asset provides an insurance against the financial capital gains or losses. As a result, the difference between $V(a, h; R^l)$ and $V(a, h; R^u)$ become less sensitive to the level of financial assets as real asset holdings increase, manifesting into a flatter curve. Thus, we can divide households in three groups, depending on their portfolio at the beginning of period $t = T$: i) households with (a, h) such that $a > a^*(h)$ and beliefs characterized by a degenerate distribution with all mass at $R^* = R^l$ (trusting households); (ii) households with (a, h) such that $a < a^*(h)$ and beliefs characterized by a degenerate distribution with all mass at $R^* = R^h$ (skeptical households); (iii) households with (a, h) such that $a = a^*(h)$ will be indifferent about any future choices by the monetary authority and have indeterminate beliefs (indifferent households).

Figure 7: Winners and losers from policy: $\boldsymbol{V(a,h,R^l) - V(a,h,R^u)}$



Notes: The figure plots the difference in continuation values at $t = T$, $V(a, h, R^l) - V(a, h, R^u)$, as a function of a on the horizontal axis for three different values of h, using the parameter values in Table 3.

The solution to the household problem at $t \geq T$ cannot be generally characterized by first order conditions because of the kink in expected continuation value at $a = a^*(k)$ due to the shift in worst-case beliefs. We can however use the MaxMin theorem to write the household problem at $t = 0$ as follows

$$\min_{G \in \mathcal{P}(\Omega)} \Big\{ \max_{\{c_{xt}, l_{xt}, a_{xt+1}, h_{xt+1}\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} \hat{\beta}^t \, U(c_{xt}, l_{xt}) \; + \hat{\beta}^T \int_1^{\bar{\omega}} V\left(a_{xT}, h_{xT}, R^*\right) G(dR^*) \Big\}.$$

For given beliefs G the household problem is continuously differentiable and the first order conditions in equations (11), (34) and (35), together with the budget constraint in (28), characterize its solution. Given the optimal allocations for given beliefs, we can then choose the worst-case beliefs that minimizes the household value at $t = 0$.

The firm problem The firm problem at $t = 0$ is similar to the household's one, but simpler to characterize given that firms are homogeneous and will have the same worst-case beliefs. As households, firms will also learn at $t = T$ about the path of future nominal interest rates, so that they face ambiguity about future policy only at $t < T$. Each firm i at $t = 0$ chooses the path of prices $\{p_{it}\}_{t=0}^{T-1}$ to maximize expected discounted real profits,

$$W_0\left(p_{i-1}\right) = \max_{\{p_{it}\}_{t=0}^{T-1}} \Big\{ \sum_{t=0}^{T-1} m_{0,t} \, d_t(p_{it}, p_{it-1}) + m_{0,T} \min_{G \in \mathcal{P}(\Omega)} \int_{\Omega} W\left(p_{iT-1}, R^*\right) \, G(dR^*) \Big\},$$

where $d_t(p_{it}, p_{it-1}) \equiv (p_{it}/p_t - w_t)(p_{it}/p_t)^{-\theta} Y_t - \kappa(\pi_{it}, Y_t)$ denote firm's profit, subject to the normalization $p_{i-1} = 1$ for all i. We assume that firms face the same ambiguity of households, $\Omega = [R^l, R^u]$. The function $W(p, R^*)$ denotes the equity value of the firm at the beginning of period T, as defined in (37). As in the case of perfect foresight, firms set the same price in each period as they face the same problem. Optimal firm pricing implies that equation (38) also holds at $t \le T$, with firms' expectations determined by their worst-case belief. At our parameter values, the firm equity value $W\left(p, R^*\right)$ is decreasing in the nominal interest rate R^*, because an expansionary monetary policy inflates the equity value expressed in real terms.[8] Therefore, firms will make their pricing decisions under a worst-case probability distribution that assigns all probability mass to the event that the monetary authority charges the highest interest rate in the support of realizations considered possible, i.e. $R^* = R^u$.

Equilibrium We can characterize the equilibrium of the economy in two different time periods: i) before the ambiguity is resolved at $t < T$; and ii) after the ambiguity is resolved at $t \ge T$. Solving for T requires solving for a fixed point problem. At any $t \ge T$ agents have perfect foresight on the path of the economy, and its equilibrium is described by Lemma

[8] At our calibration, the profits are decreasing to an expansionary monetary shock, due to the price adjustment costs, but less than the fall in the real rate, so that the present discounted value of profits goes up.

4. At $t < T$ agents "agree to disagree" about the equilibrium path of quantities and prices at $t \geq T$, but face and agree on the same equilibrium prices at $t < T$. Once we have aggregate demand and inflation from the solution to the households and firms problems we can compute $\hat{R}_t$ at $t \leq T$ and verify that T is the first date at which the monetary authority would like to deviate from its announcement, i.e. $\hat{R}_t = 1$ for $t < T$ and $\hat{R}_t > 1$ for $t = T$.

5 Calibration

The model is calibrated at quarterly frequency. We next describe our calibration strategy. Table 3 collects the parameter values and targets used in our baseline calibration.

Preferences and technologies We set β to match a share of financial income over total income of 15%, which is in line with micro level data from the euro-area Household Finance Consumption Survey (HFCS). We obtain a steady state return on savings of 6%, which is the approximate real return from investing in the stock market in the euro area. The elasticity of intertemporal substitution (EIS) is set to 0.5 and the Frisch elasticity of labor supply to 2, which are in the range of values commonly used in the literature; see Guvenen (2006) and Keane and Rogerson (2012). The parameter governing the elasticity of substitution across varieties θ is set to target a steady state labor share of 2/3. The resulting value, $\theta = 3$, is in the range of values typically used in macro models, albeit on the lower end.[9] The parameter governing the cost of price adjustment κ_0 is used to match the elasticity of inflation to current marginal cost in the Phillips curve θ/κ_0, which we set at a value of 0.1, quite closely in line with the literature (see Schorfheide 2008). We normalize steady-state labor supply to one, which determines the scaling factor of the utility function ψ_0. The parameter of the Taylor rule in (3) is set to the standard value $\phi = 1.5$. The parameter governing the adjustment cost function of real assets, χ_0, is set so that the ratio between adjustment costs and the value of transactions is on average equal to 1% in the six quarters after the announcement, in line with the average value of adjustment costs for real assets estimated by Alvarez, Guiso, and Lippi (2012).

The initial steady state distribution of assets We parametrize the joint distribution of financial and real assets over a support of $n = 1000$ discrete points, of equal mass $(a_1, h_1), (a_2, h_2), \ldots (a_n, h_n)$. Each a_i corresponds to a per mile of the distribution of euro-area net financial assets (FA), scaled by the average yearly household labor income. For each per mile of the distribution of FAs a_i we calculate the associated average value of

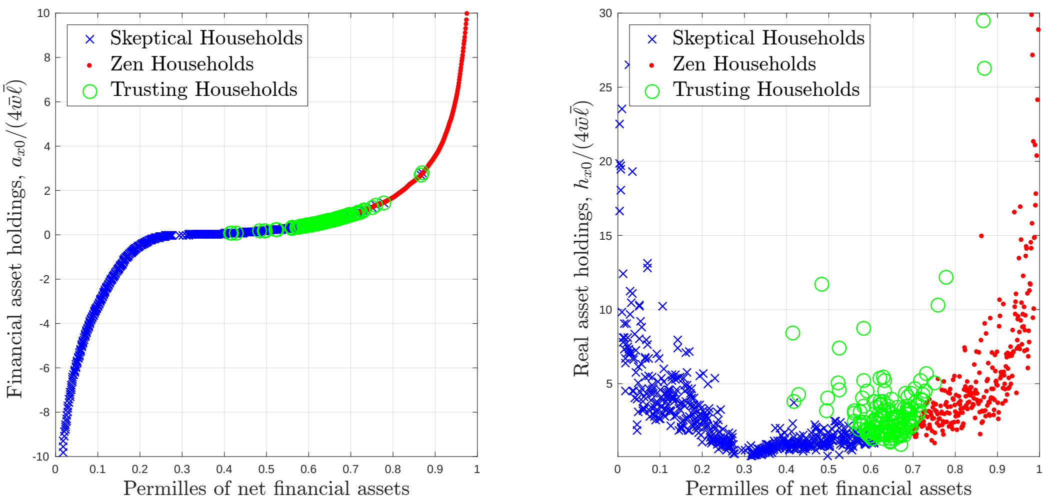
[9] For instance, Midrigan (2011) assumes $\theta = 3$, as we do. Instead Golosov and Lucas (2007) work with $\theta = 7$, which in our model would yield a labor share greater than 2/3, while in reality the labor share has fallen below 2/3 over the last decade; see Karabarbounis and Neiman (2014).

Table 1: Baseline Calibration

Model		Data	
Parameter	Value	Moment	Value
β	0.985	Yearly Euro area real stock market return	0.06
σ	2	Elasticity of intertemporal substitution	0.5
ψ	0.5	Frisch elasticity of labor supply	2
ψ_0	0.66	Labor supply normalization in steady-state	1
θ	3	Labor share	0.66
κ_0	30	Slope of the Phillips curve	0.1
χ_0	0.5	Mean real asset adjustment cost to value of transaction	0.01
Υ	0.25	Mean net financial assets to yearly average labor income	2
ϕ	1.5	Taylor rule response to inflation	1.5
$\hat{\beta}$	1.005	Percentage fall in Euro area GDP in 2012	2%
t_β	6	Expected length of the Euro area 2012 recession	6
t_r	10	Peak in response of forward rates to FG	10
R^h	1.0125	Average inflation expectation response to FG	10 bp
R^l	1.0075	Max differential in inflation expectation response to FG	20 bp

real assets h_i. Data on FAs and RAs are from HFCS. FA is the difference between total financial assets (deposits, bonds, mutual funds, voluntary pension funds) and total financial liabilities (mortgages plus non-mortgage debt). RA are the sum of all real estate properties of the household other than her main residence, valuables, and self-employment businesses; see the Appendix for further details. We set Υ to match a ratio of 2 between aggregate households' initial assets and yearly labor income, which corresponds to the value found in the HFCS. Panel (a) of Figure 8 plots the per mile of the distribution financial assets a_i, scaled by average yearly labor income. Panel (b) plots the value of real assets associated to each per mile of distribution of FAs h_i. The standard deviation of the distribution of FAs is high, equal to 23, and skewness is substantial, at 116. The different colors capture the model predicted relationship between the households' initial financial asset position and their beliefs about the implementation of the forward guidance announcement. We distinguish three groups of households. About 77% of households are characterized by worst-case beliefs that will assign probability one to $R_t = R^h$. These are the households with low enough financial asset holdings (denoted in blue). About 18% of households are characterized by worst-case beliefs that will assign probability one to $R_t = R^l$. These are the households with high enough financial asset holdings at $t = 0$ (denoted in red). The remaining households (denoted in green) are indifferent about the implementation of the announcement and characterized by an initial level of financial asset that is the neighbor of the threshold $a^*(h)$, which we saw before being about two times yearly labor income.

Figure 8: The distribution of net financial assets from HFCS



(a) Permilles of households' FAs

(b) Real assets for permilles of FAs

Notes: Panel (a) plots the per-miles a_i of the distribution financial assets of European of households; the y-axis has been truncated to the left and to the right for illustrative purposes. Panel (b) plots the average value of real assets associated to each per mile of the distribution of financial assets, again scaled by average annual labor income.

Liquidity trap and forward guidance announcement We set the discount factor over the period $[0, t_\beta]$ to $\hat{\beta} = 1.005$ so that, in absence of the monetary policy announcement, output is on impact at the time of the shock 2% below steady state, roughly in line with the fall of Euro area GDP in 2012. The liquidity trap is assumed to last six quarters, $t_\beta = 6$, to match the expected duration of the recession in the Euro area as predicted on average by the Survey of Professional Forecasters (SPF) in 2013, before the forward guidance announcement on July 2013. We set $t_r = 10$ to match the peak in the response of instantaneous forward rates after the forward guidance announcement, see Coeuré (2013), ECB (2014), and Picault (2017).

Ambiguity set To quantify the (possible) increase in disagreement among European households about expected future inflation after forward guidance, we rely on micro level evidence for Italy.[10] We interpret self reported inflation expectations as measuring the beliefs about future inflations on the basis of which individuals act (say about the inflation that arises under the nominal interest rate ω that solves the maxmin problem of the household).[11] The Appendix fully describes the source and construction of the variables used. The data are quarterly and the sample covers the period 2012:I-2014:II. The end of the sample is dictated by the start of the ECB's Quantitative Easing program in 2015:I. For each Italian province we calculate the pre-announcement (in 2012) fraction of households with positive Net Financial Assets (creditor households). Expected inflation is measured two quarters ahead. In each province i and quarter t, we calculate the following measure of the (average) inflation expectation bias of agents in the province

$$\hat{\pi}_{it} \equiv E_{it}[\pi_{it+2}] - \pi_{it+2}, \tag{42}$$

where $E_{it}[\pi_{it+2}]$ and π_{it+2} are expected inflation and realized future inflation, respectively. To evaluate whether, in response to forward guidance, the inflation expectations have increased more for creditor households than for debtor households, we run the following Difference-in-Differences regression:

$$\hat{\pi}_{it} = \overline{\phi} F_i + \phi F_i \times \mathbb{I}_{t \geq t_0} + \beta X_{it} + \epsilon_{it} \tag{43}$$

where F_i is equal to the (standardized) proportion of creditor households in the province. The controls X_{it} includes a full set of time and province dummies. $\mathbb{I}_{t \geq t_0}$ is a dummy equal to one in the quarter of the announcement (t_0=2013:III) and in all subsequent quarters, zero in previous quarters. The coefficient $\overline{\phi}$ measures the average effect of F_i on inflation expectations. The Difference-in-Differences coefficient ϕ measures the increase in the effect

[10]To check robustness, in the Appendix we also report country level evidence for the Euro Area. For countries we do not have information on expected inflation but just on the fraction of households who think that inflation will increase in the next year relative to the past year.

[11]Hurd (2009) discusses the empirical evidence supporting the claim that the subjective probabilities self reported by households explain well their behavior. The evidence indicates that self reported expectations are biased and heterogeneous across households but they have strong power in predicting household's behavior. See Kézdi and Willis (2011) for evidence that households' subjective beliefs about future stock market returns explain stock market investment and Armantier, de Bruin Wändi, Topa, van der Klaauw, and Zafar (2015) for specific evidence about the effects of self-reported inflation expectations on households' financial investment decisions.

of F_i on inflation expectations in the quarters after the announcement. The results from estimating (43) are reported in Table 2, which indicates that the inflation expectations have become more correlated with the financial position of households. After the ECB

Table 2: Effects of Forward Guidance on expected inflation, Micro Evidence

VARIABLES	$\hat{\pi}_{it}$
Announcement-dummy $\times$ F_i (coefficient ϕ)	.10***
	.04
Effect of financial position F_i (coefficient $\overline{\phi}$)	.02
	.02
R-squared	.35
No. of observations	1082
No. of i units	110

Notes: Results from regression (43). The regression includes year and individual fixed effect. The dependent variable is $\hat{\pi}_{it}$, in (43). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

announcement, provinces with two-standard-deviations more of creditor households—which represents two thirds of the cross sectional variation— experience an increase of around 20 basis points in their inflation expectations. We take this as a (conservative) target for the maximum increase after forward guidance in the disagreement about expected inflation between creditor and debtor households. To measure changes in average expected inflation we rely on Inflation Linked Swaps (ILS) which measure directly the market's expected inflation rate.[12] The data indicate that in response to forward guidance expected inflation has increased by around 10 basis points at a time horizon of two years which is in line with the evidence by Andrade and Ferroni (2016). We take this as a second target to calibrate the announcement. The two target identifies the interval of the possible realization of nominal interest rates after T, $[R^l, R^h]$, which fully characterizes the effects of forward guidance in the model.

6 Quantitative results

We obtain an exact solution of the model by global non-linear methods (see the Appendix for details). Figure 9 plots the impulse responses to the forward guidance announcement

[12] An ILS is a contract, which involves an exchange of a fixed payment (the so-called 'fixed leg' of the swap) for realised inflation over a predetermined horizon.

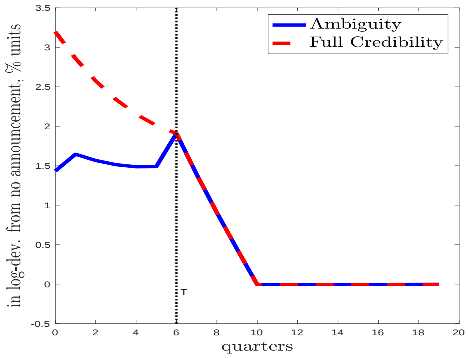
of selected economic variables, expressed in deviations from the corresponding equilibrium values in the counterfactual case of no policy intervention displayed in Figure 5. Thus the impulse responses isolate the effect of the monetary policy announcement on the economy. The solid blue lines correspond to our baseline model calibrated as of Table 3. The red dashed lines correspond to a counterfactual with no ambiguity about the credibility of the monetary policy announcement where all firms and households believe that the monetary policy will be at the lower bound of the ambiguity set, i.e. $R_t = R^l$ for $t \in [t_\beta + 1, t_r]$. This counterfactual is useful to identify the contribution of ambiguity for the effects of the announcement. The vertical black line pinpoints the time of the resolution of ambiguity announcement, T, which happen to coincide with the time of exiting the liquidity trap, $t = t_\beta + 1$. For simplicity of exposition, we assume that the announcement that is then implemented at $t = T$ corresponds to $\omega = R^l$, but this is without loss of generality since the focus is on the response of the economy in the interim before T which only depends on the beliefs at $t < T$ about ω and not on its ex-post realization.

At $t \geq T$, nominal and real interest rates fall owing to the monetary expansion (see panels (c) and (d)), which leads to an expansion in output (panel (a)) and positive inflation (panel (b)). The effects in our model with ambiguity and in the alternative full-credibility benchmark are identical, which follows from Proposition 7.

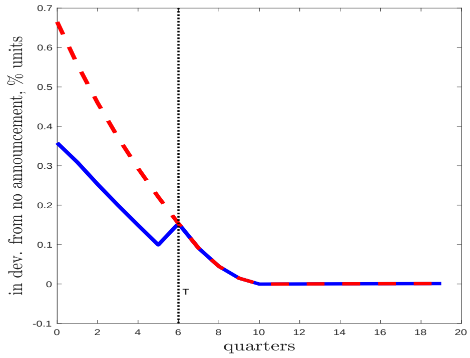
At $t < T$, the two economies behave very differently: in the baseline model with ambiguity output and inflation rise substantially less than in the full-credibility benchmark. Quantitatively, in the six quarters before implementation the baseline model predicts an output gain that slightly more than half of the effect predicted in the full-credibility benchmark, on average 1.3% and 2.5% higher output with respect to steady state respectively. Similarly, the response of quarterly inflation at $t < T$ is on average 0.21% in our model against 0.38% in the full-credibility benchmark, explaining the different behavior of the real rate in this period.

The different beliefs that agents have in the two economies about the path of the nominal interest rate at $t \geq T$ explain the different responses of output and inflation at $t < T$ to the announcement. Figure 10 plots the responses to the forward guidance announcement of agents' expectations at time $t = 0$ about the path of selected variables at $t \geq 0$ on the horizontal axis. Agents are divided in two subsets: i) the blue solid line plots the expectations of agents believing that the nominal rate will be relatively lower, $R_t = R^l$ for $t \in [t_\beta + 1, t_r]$, labeled as *trusting*; ii) the red dashed line plots the expectations of agents believing that the nominal rate will be relatively higher, $R_t = R^h$ for $t \in [t_\beta + 1, t_r]$ labeled as *untrusting*. We recall that Figure 8 provides the mapping from households' portfolios at $t = 0$ and these two subsets of agents, and that firms act as *untrusting* agents as they would gain from an expansionary policy. By construction there is no heterogeneity

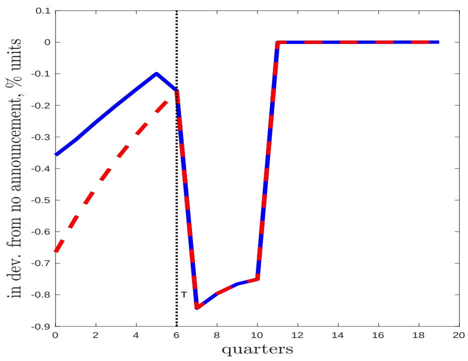
Figure 9: Impulse responses to the forward guidance announcement



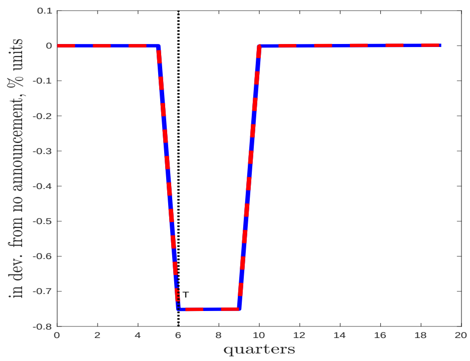
(a) Output Y



(b) Inflation π



(c) Real interest rate r

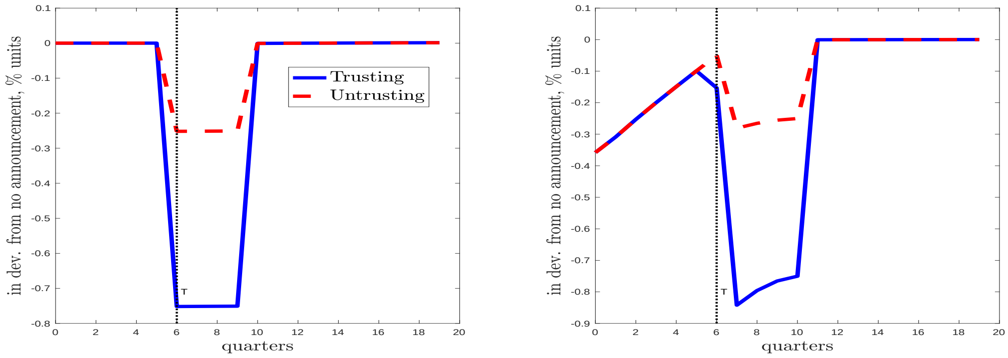


(d) Nominal interest rate R

Notes: The blue solid lines plot the responses of selected variables to the forward guidance announcement predicted by our model calibrated as of Table 3. The responses are displayed in deviation from the case of no announcement shown in Figure 5. The red dashed lines refer to the counterfactual case where we assume that all agents believe that $R_t = R^l$ for $t \in [t_\beta + 1, t_r]$. The vertical black line pinpoints the time of the resolution of ambiguity announcement, T.

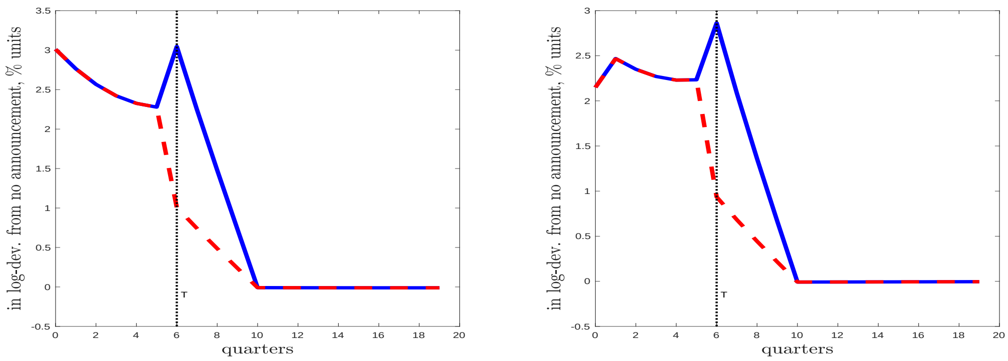
in agents' worst-case beliefs on the path of economic variables at $t < T$. The agents instead make economic decision on the basis of different beliefs about the equilibrium of the economy at $t \geq T$. In particular, *trusting* agents act on the basis of expectations of lower nominal interest rates and, as a consequence, lower real interest rate, higher labor income and higher real asset price than *untrusting* agents. Relatively to the full-credibility benchmark, the distribution of households and firms beliefs affect the response of output and inflation at $t < T$ through three main channels. First, as *trusting* households are the ones with positive exposure to financial capital income, the lower expected real interest rate at $t \geq T$ has a negative income effect on their demand at $t < T$, thus reducing aggregate demand. Second, as *untrusting* households expect lower labor income at $t \geq T$ they will also demand less at $t < T$ relatively to the full-credibility benchmark. Third, as firms

Figure 10: Heterogeneous beliefs about the dynamics of the economy



(a) Nominal interest rate R

(b) Real interest rate r



(c) Real asset price q

(d) Labor income $w\ell$

Notes: The figure plots the responses to the forward guidance announcement of agents expectations at time $t = 0$ about the realization of selected variables at $t \geq 0$ on the horizontal axis. The blue solid line plots the expectations of agents believing that $R_t = R^l$ for $t \in [t_\beta + 1, t_r]$. The red dashed line plots the expectations of agents believing that $R_t = \bar{\omega}$ for $t \in [t_\beta + 1, t_r]$. The model is calibrated as of Table 3. The responses are displayed in deviation from the case of no announcement shown in Figure 5.

are *untrusting* agents they expect relatively lower inflation at $t \geq T$ and, because of the forward-looking pricing behavior, will have lower price inflation at $t < T$ causing relatively higher real rates and lower aggregate demand in this period.

As a result of the heterogeneous beliefs about the returns on financial and real assets *trusting* households reduce their holdings of bonds and increase their holdings of real assets, as the real asset offers an insurance against the expected lower capital income from bonds associated to a more expansionary monetary policy. On the other side, *untrusting* households are willing to give away real assets as their price is too high according to their beliefs of relatively higher real rates at $t \geq T$. The presence of adjustment costs in real assets prevents a full reallocation of real assets from *untrusting* to *trusting* households. Ta-

Table 3: Evolution of households portfolio allocation after the announcement

	Trusting		**Untrusting**	
	$t = 0$	$t = T$	$t = 0$	$t = T$
Mean portfolio allocation share to bonds	0.66	0.59	-0.89	-0.52
Dispersion of portfolio allocation share to bonds	4.62	4.28	4.40	3.39

Notes: The table provides statistics about the share of household wealth invested in the financial asset, defined as $a_{xt}/(a_{xt} + q_t\, h_{xt})$, computed at the beginning of periods $t = 0$ and $t = T$. Statistics are computed for two subsets of the populations: i) households who believe $R_T = R^l$ (trusting), and ii) households who believe $R_T = \bar{\omega}$ (untrusting).

ble 3 reports statistics about the portfolio rebalancing of the different types of households. Trusting households have an initial share of their wealth invested in financial asset equal to 66% on average, which gets reduced to 59% at the beginning of period T. On the contrary, untrusting households are on average net financial debtors, with their debt position equal to 89% of their net wealth on average in the initial steady state. Their indebtedness is substantially reduced to 52% of their total wealth. We notice that the reduction in the relative indebtedness of *untrusting* households happens both because these agents sell real assets to pay back the debt and because the value of the real asset that they keep holding increases.

7 Robustness and extensions

We now discuss some robustness exercises. TBC

7.1 Government debt into households liabilities

8 Conclusions

We have characterized the equilibrium of a new Keynesian model in which ambiguity-averse households with heterogeneous net financial wealth use a worst-case criterion to judge the credibility of monetary policy announcements. An announcement of monetary loosening is less expansionary in our framework than under full credibility, and it can even be contractionary when the inequality in wealth is sufficiently pronounced. This is because wealthy creditor households are more prone to believe the announcement of loosening than poor, indebted households. Hence there is a fall in perceived aggregate wealth, which if large enough causes a contraction in aggregate demand. To gauge the importance of this mechanism, we have considered the start of forward guidance by the ECB in July 2013. Calibrating the model to match the entire distribution of European households' net financial wealth, we find that forward guidance is fifty per cent less expansionary than in the full credibility benchmark. We have analyzed the effects of monetary policy announcements, but the same logic would apply to announcements about any future policy that, if implemented, would generate winners and losers, such as pension reform, or revisions to competition, innovation or fiscal policy, or changes to labor market institutions like unemployment insurance and job protection. Generally, the announcements of future reforms that will redistribute wealth if implemented, tend to have little and sometimes even unintended perverse effects when agents are ambiguity-averse, because the net losers tend to give more credit to announcements than the net winners.

Throughout the analysis, we have maintained some simplifying assumptions that it would be interesting to relax in future research. For example, we allowed households to trade just in a one-period bond with a predefined nominal interest rate and some real assets. In practice financial markets allow households to buy a variety of assets. All this is relevant because the effects of monetary policy on the real return on investment could differ across assets, which would imply that monetary policy losers and winners are not perfectly identified by the sign of their net financial asset position. Allowing households to face a more complex portfolio problem might generate further insights into the interaction between redistribution and ambiguity aversion. Moreover, in our model, households cannot trade *real* interest rate swaps, which would insure them against future changes in monetary policy. This assumption is realistic, because the market for real interest rate swaps is tiny and only a very few financially sophisticated households hold swaps (Lusardi and Mitchell 2014). Yet real interest rate swaps would allow households to exploit trade opportunities induced by differences in their beliefs and would generally increase the effectiveness of monetary policy announcements.

In our model we have also abstracted from the role of fiscal policy in the transmission of monetary policy and from heterogeneity in households' marginal propensity to consume, ambiguity, and income. Both these issues are important and would interact with our mechanism. For example Kaplan, Moll, and Violante (2016b) emphasize that monetary policy has an impact on fiscal transfers, which in turn affect households' disposable income and hence consumption and aggregate demand. But in our model, fiscal transfers, and in particular their timing, would also affect the formation of households' beliefs, so governments could use them strategically to enhance the credibility of monetary policy. Finally, in our model households differ only in initial financial wealth, but in reality households also differ in marginal propensity to consume (Werning 2015), degree of ambiguity aversion (Dimmock, Kouwenberg, Mitchell, and Peijnenburg 2016), labor income, and human capital. Under ambiguity aversion, this heterogeneity has a first-order effect on the formation of households' beliefs and thereby on the effect of policy announcements.

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APPENDIX

Section A contains some proofs and theoretical derivation, Section B describes the data, Section C discusses computational details, and Section D provides further details on the robustness exercises in Section 7.

A Theoretical derivations

This section contains of the proofs of some results stated in Section 3, the full characterization of the formulation of the model in Section 3 when we allow (i) the government to intervene with some transfer, (ii) agents to trade in some real assets, (iii) households to be averse ambiguity according to the formulation proposed by Hansen and Sargent (2001, 2008).

A.1 Proofs of results in Section 3

Proof of Lemma 2. $R_0 = \bar{R}$ because the economy is initially in a steady state. Given the timing of the monetary announcement, prices do not respond at $t = 0$ so $\Pi_0 = \Pi_0^* = 1$, which given (3) yields $R_1 = \bar{R}$. Lemma 1 implies that the economy is back to steady state starting from $t = 1$ so it must be that $r_t = \bar{R}\ \forall t \geq 2$. By assumption we also have $\Pi_t = \Pi_t^* = 1$, $\forall t \geq 2$ so we have $R_t = r_t = \bar{R}\ \forall t \geq 2$, which immediately gives $R_t = \bar{R}\ \forall t$. And this together with (3) also implies that $\Pi_t = \Pi_t^*\ \forall t \geq 0$. ■

Proof of Proposition 1. Under full credibility, household $j = c, d$ solves the problem

$$\max_{\{c_{js}, l_{js}, a_{js+1}\}_{s\geq 0}} \sum_{s=0}^{\infty} \beta^s U(c_{js}, l_{js}),$$

subject to the budget constraint in (2). The first order condition for the consumption choices of household j at $t = 0$ yields the Euler condition

$$\left(c_{j0} - \psi_0 \frac{l_{j0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \beta\, r_1 \left(c_{j1} - \psi_0 \frac{l_{j1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{44}$$

where $r_1 = \bar{R}\,\varepsilon^{-1}$, which uses full credibility and Lemma 2. Output Y_0 can be obtained using the market clearing condition for final consumption

$$Y_0 = \frac{c_{c0} + c_{d0}}{2},$$

where, given Lemma 1, c_{j0} and c_{j1} should satisfy

$$c_{j0} = Y_0 + \bar{R}\, a_{j0} - a_{j1} \quad \text{and} \quad c_{j1} = \bar{Y} + (\bar{R} - 1)\varepsilon^{-1}\, a_{j1} \quad \forall j = c, d. \tag{45}$$

We can substitute (45) into (44), and use the conditions for financial market clearing at

$t = -1$, $a_{c0} = -a_{d0} = B$, and at $t = 0$, $a_{c1} = -a_{d1} = B'$. Since $l_{jt} = Y_t$, we obtain that (44) evaluated for $j = c$ and for $j = d$ is equal to conditions:

$$\frac{\bar{N} + (\bar{R} - 1)\varepsilon^{-1} B'}{N_0 + \bar{R}B - B'} = \varepsilon^{-\frac{1}{\sigma}} \tag{46}$$

and

$$\frac{\bar{N} - (\bar{R} - 1)\varepsilon^{-1} B'}{N_0 - \bar{R}B + B'} = \varepsilon^{-\frac{1}{\sigma}}, \tag{47}$$

respectively. After solving for N_0, we obtain

$$N_0 = \bar{N}\varepsilon^{\frac{1}{\sigma}}, \tag{48}$$

which can be substituted into (46) to solve for B' to obtain

$$B' = \frac{\bar{R}\,B}{(\bar{R} - 1)\,\varepsilon^{\frac{1}{\sigma}-1} + 1} > 0.$$

This means that $B'/\varepsilon - B < 0$ if $\varepsilon > 1$, and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$, which completes the proof. ■

Proof of Proposition 3. $F(a_{j0}, a')$ is continuous in a' and its derivative, $F_2(a_{j0}, a')$, is globally strictly decreasing in a' with a discontinuity point at $a' = 0$. So $F(a_{j0}, a')$ is concave in a', which guarantees a unique solution to the problem in (16). Moreover we have that the marginal value of a household's savings, a', is strictly increasing in her beginning-of-period wealth a_{j0}:

$$F_{12}(a_{j0}, a') = \sigma\bar{R}\left(N_0 + \bar{R}a_{j0} - a'\right)^{-\sigma-1} > 0. \tag{49}$$

■

Proof of Lemma 3. Panel (b) of Figure 3 implies that a credit crunch requires that $\forall j = c, d$, the following conditions should hold:

$$F_2^-(a_{j0}, 0) = -\left(N_0 + \bar{R}a_{j0}\right)^{-\sigma} + \frac{\beta\bar{V}'(0)}{\min(1, \varepsilon)} > 0 \tag{50}$$

$$F_2^+(a_{j0}, 0) = -\left(N_0 + \bar{R}a_{j0}\right)^{-\sigma} + \frac{\beta\bar{V}'(0)}{\max(1, \varepsilon)} < 0 \tag{51}$$

where we used the expression for individual beliefs in (15). Given (A.3), the condition $a_{c0} = -a_{d0} = B$ and doing some simple algebra it is confirmed that inequalities (50) and (51), evaluated both at $j = c$ and at $j = d$, are equivalent to the condition $N_0^A < N_0^B$, or alternatively that the inequality in (20) fails. ■

Proposition 8 (Steady state imbalances) *After an inflationary announcement $\varepsilon > 1$,* *the new steady state financial imbalances after implementation, B'/ε, always decrease ($B'/\varepsilon < B$), and they decrease more than in the full-credibility benchmark. After a deflationary announcement $\varepsilon < 1$, there are two (strictly positive) thresholds $\tilde{B}_1$ and $\tilde{B}_2$, with $\tilde{B}_1 < \tilde{B}_2$, such that for $B < \tilde{B}_1$, B'/ε falls; for $B \in [\tilde{B}_1, \tilde{B}_2]$, B'/ε increases, but less than in the full-credibility benchmark; and for $B > \tilde{B}_2$, B'/ε increases, and more than in the benchmark.*

The effects on the end-of-period imbalances B' depend on who stands to gain from the redistribution of expected wealth. To see this, observe that (**??**) implies that B' is a function of the relative consumption of debtors $j = d$ and creditors $j = c$:

$$B' = \frac{c_{d0} - c_{c0}}{2} + \bar{R}B.$$

When $\varepsilon > 1$, debtors do not believe the announcement (see Propositions 4 and 6) so their relative consumption c_{d0} increases less then under full credibility, which generally makes B' smaller than under full credibility, independently of B. When $\varepsilon < 1$ and B is small, the zen effect (induced by the kink in the value of households' savings) leads to credit crunches (see Proposition 6); and it makes the end-of-period imbalances B' smaller than in the full credibility benchmark. If $\varepsilon < 1$ and B is large, creditors do not believe the announcement and neglect the large wealth gain associated to $\varepsilon < 1$ (again following from Propositions 4 and 6), so c_{c0} is (relatively) smaller, hence B' larger, than under full credibility. This leads to the unintuitive result that when B is large enough and $\varepsilon < 1$, ambiguity aversion induces larger imbalances.

Proof of Proposition 8.

The proof proceeds in three steps. We characterize (i) the full-credibility (FC) benchmark ($\hat{\tau} = 1$ and $\rho = 0$), (ii) an inflationary announcement $\varepsilon > 1$, and (iii) a deflationary announcement $\varepsilon < 1$.

FC benchmark The properties of the FC benchmark are given in Proposition 1, which implies that $N_0 = \varepsilon^{\frac{1}{\sigma}}\bar{N}$; $B' > 0$; $B'/\varepsilon - B < 0$ if $\varepsilon > 1$; and $B'/\varepsilon - B > 0$ if $\varepsilon < 1$. When $\hat{\tau} = 1$ and $\rho = 0$, (SA) also implies that

$$B'/\varepsilon - B = \frac{\varepsilon^{-\frac{1}{\sigma}} N_0 - \bar{N} + B\left[\varepsilon^{-\frac{1}{\sigma}}\left(\bar{R} - \varepsilon\right) - \left(\bar{R} - 1\right)\right]}{\bar{R} - 1 + \varepsilon^{1-\frac{1}{\sigma}}} \tag{52}$$

with $N_0 = \varepsilon^{\frac{1}{\sigma}}\bar{N}$.

Case $\varepsilon > 1$ If (20) fails we have a credit crunch equilibrium, $B'/\varepsilon = 0$, which immediately implies a larger fall in B'/ε than in the FC benchmark. If (20) holds, then $B' > 0$ and, from Proposition 4, we have $\bar{\tau} = 1/2$ and $\rho = 1$, which can be substituted into (SA) to show that B'/ε still satisfies (52). After substituting $\bar{\tau} = 1/2$ and $\rho = 1$ into (19) we obtain

$$N_0 = \bar{N}\,(\omega\,\varepsilon^{\frac{1}{\sigma}} + 1 - \omega) - B\,\zeta\,(1 - \varepsilon^{\frac{1}{\sigma}-1}) < \bar{N}\,\varepsilon^{\frac{1}{\sigma}},$$

which, together with (52), proves in general that, $\forall B$, B'/ε falls more than in the FC benchmark.

Case $\varepsilon < 1$ Proposition 6 implies that if (20) fails, we have a credit crunch equilibrium, $B'/\varepsilon = 0$. If (20) holds, $B'/\varepsilon > 0$, and, from Proposition 4, we have $\bar{\tau} = 1/2$ and $\rho = -1$, which can be substituted into (SA) and (19) to obtain

$$B'^{-1}\frac{N_0 - \bar{N}}{\bar{R}} + (\varepsilon^{-1} - 1)\, B,$$

and

$$N_0 = \bar{N}\,(\omega + (1-\omega)\,\varepsilon^{\frac{1}{\sigma}}) + B\,\zeta\,(1 - \varepsilon^{\frac{1}{\sigma}-1}).$$

By combining the two expressions, we conclude that $B'/\varepsilon - B < 0$ if

$$B < \tilde{B}_1 \equiv \frac{\bar{N}\,(1-\varepsilon^{\frac{1}{\sigma}})}{2\,\bar{R} - (\bar{R}-1)\,\varepsilon^{\frac{1}{\sigma}} - \varepsilon\,(1+\bar{R})}, \tag{53}$$

where $B = \tilde{B}_1$ satisfies (20), which generally implies that, $\forall B < \tilde{B}_1$, B'/ε falls. Moreover, (DA) evaluated at $\bar{\tau} = 1/2$ and $\rho = -1$ yields

$$B'/\varepsilon - B = \frac{\bar{N} - \varepsilon^{-\frac{1}{\sigma}}\,N_0 + B\,\left[\varepsilon^{-\frac{1}{\sigma}}\left(\bar{R} - \varepsilon\right) - (\bar{R}-1)\right]}{\bar{R} - 1 + \varepsilon^{1-\frac{1}{\sigma}}}. \tag{54}$$

Comparing (52) with (54), we immediately conclude that B'/ε increases less than in the FC benchmark if and only if $N_0 > \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$. Under $\varepsilon < 1$ and $\rho < 0$, which is implied by Proposition 4), N_0 is strictly decreasing in B, whereas N_0 in the FC benchmark ($\rho = 0$) is invariant to B. So we conclude that B'/ε increases less (more) than in the FC benchmark if and only if $B < \tilde{B}_2$ ($B > \tilde{B}_2$) where

$$\tilde{B}_2 \equiv \bar{N}\,\frac{[1 + (\bar{R}-1)\varepsilon^{\frac{1}{\sigma}-1}]\,(1-\varepsilon^{\frac{1}{\sigma}})}{\bar{R}\,(\bar{R}-1)\,(1-\varepsilon^{\frac{1}{\sigma}-1})}$$

is the value of B at which $N_0 = \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$. Remember that at $B = \tilde{B}_1$ we have $B'/\varepsilon - B = 0$, so from (54) we conclude that $N_0 > \bar{N}\,\varepsilon^{\frac{1}{\sigma}}$. Thus the definition of $\tilde{B}_2$ together with the fact that N_0 is strictly decreasing in B immediately implies that $\tilde{B}_2 > \tilde{B}_1$. ■

A.2 The model of Section 3 with real assets

There is a real asset in fixed supply (say a Lucas tree) with

$$\frac{h_{c0} + h_{d0}}{2} = H. \tag{55}$$

where h_{j0} denotes the initial endowment of real assets for households of type $j = c, d$ and H is the aggregate amount of real assets available.

A unit of the real asset yields a return $\left(\bar{R}-1\right)$ per period for sure where Suppose $\bar{R}=\beta^{-1}$.. Adjusting the holding of the real asset from h to $h^{'}$ involves convex adjustment costs $X\left(h'-h,h\right)$ homogenous of degree one in $h'-h$ and h with $X_1, X_{11}\geq 0$ and $X(0,h)=X'(0,h)=0$. I practice we will be assuming that $X\left(h'-h,h\right)=\frac{x_0}{2}\left(\frac{h'-h}{h}\right)^2 h$. We denote by

$$N^H(Y,H)\equiv Y-\psi_0\frac{Y^{1+\psi}}{1+\psi}+\left(\bar{R}-1\right)H$$

the output net of the effort cost of working of an economy with H available units of the real assets. We normalize the supply of the real asset to $H=1$ and choose h_{d0}, h_{c0}, and ψ_0 (while leaving all other quantities unchanged) so as to guarantee that in the initial steady state the consumption level of creditors and debtors is unchanged relative to the baseline model—and that the market for the real asset clears as in (55). This requires having $h_{d0}=h_{c0}=H$ and setting ψ_0 so that

$$\bar{N}\equiv N(\bar{Y})=N^H(\bar{Y},H) \tag{56}$$

where $\bar{N}\equiv N(\bar{Y})$ denotes steady state net output in the baseline model. We denote by q_t the price of the real asset at time t. In steady state we have $q=1$. Notice that in period one we are back to steady state so that $q_1=1$. All the other assumptions of the model are as in the baseline model. We denote by Δ the units of the real asset which are reallocated from one household type to the other in period zero immediately after the announcement:

$$\Delta=|h_{c1}-h_{c0}|=|h_{d1}-h_{d0}|\leq h_{d0}\mathbb{I}(h_{c1}-h_{c0}>0)+h_{c0}\mathbb{I}(h_{d1}-h_{d0}>0)$$

where $\mathbb{I}$ denotes the indicator function. We denote $X(\Delta)=X\left(\Delta,1\right)$ and $X'(\Delta)=X_1\left(\Delta,1\right)$.

The availability of the real asset allow agent to trade to exploit tradable opportunities arising from their disagreement in the expected real return of the financial asset. In fact one can show that in the absence of disagreement , the real asset is never trades. The price of the asset is adjusted to equalize with the expected return of the financial asset. In particular in the appendix we prove that Lemma **??** in the Appendix proves that in the absence of disagreement about the future expected return on the financial asset, $\rho=0$, (as for example in the full credibility benchmark) the real asset is never reallocated across households.

Proposition 9 (Equilibrium with real assets) *In the absence of disagreement about the future expected return on the financial asset, $\rho=0$, (as for example in the full credibility benchmark) the real asset is never reallocated across households. If $x_0=\infty$ we have the equilibrium of the baseline model. After any monetary announcement (either inflationary $\varepsilon>1$ or deflationary $\varepsilon<1$) the real asset is always reallocated from debtors to creditors. Output is always higher than in the baseline model the difference in output depends on the amount of the reallocation costs. When x_0 is sufficiently small the equilibrium features a credit crunch, where beliefs and housing prices are set so that $B'=0$. A Credit crunch equilibrium is more likely than in the baseline model. The equilibrium amount of net* *output, $N_0 - X(\Delta)$, is decreasing in the level of initial imbalances in the financial asset and net output contracts when B is large enough.*

We have generally proved that a real asset cannot undo our mechanism because if there are arbitrage opportunities and no adjustment costs ($x_0 = 0$ in our formulation) at best you end up in a credit crunch with $B' = 0$. This is a very important result and we should relate it to the dialysis on with Kehoe. This says that independently of whether the announcement is inflationary ($\varepsilon > 1$) or deflationary ($\varepsilon < 1$), the real asset is reallocated from debtors to creditors. This is because the expected return on the financial asset is always smaller form creditors than for debtors, so debtors are the least likely to invest in the real asset (whose return is constant and unaffected by monetary policy). (62) establishes a negatively sloped relation between the price of the real asset q_0 and the amount of reallocation, which characterizes creditors demand for housing. (63) establishes a positively sloped relation in the q_0-Δ space and can be interpreted as characterizing the supply of housing of debtors. After an inflationary announcement ($\varepsilon > 1$) creditors buy more of the real asset because their demand in (62) increases, while in response to a deflationary announcement ($\varepsilon > 1$) creditors buy more of the real asset because the supply of the real asset by debtors in (63) increases. The solution to the system in (62) and (63) determines the maximum amount of reallocation that can be sustained in the economy, which is equal to (64).

Lemma 5 *In the absence of disagreement about the future expected return on the financial asset, $\rho = 0$, (as for example in the full credibility benchmark) the real asset is never reallocated across households.*

Proof of Lemma 5. The Euler equation for the choice of financial assets for creditors a_{c1} at $t = 0$ reads as follows:

$$\left(c_{c0} - \psi_0 \frac{l_{c0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \varepsilon^{-\bar{\tau}(1+\rho)} \left(c_{c1} - \psi_0 \frac{l_{c1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{57}$$

where $\bar{\tau}$ and ρ are defined as in the baseline model. The analogous Euler equation for the choice of financial assets for debtors a_{d1} is as follows:

$$\left(c_{d0} - \psi_0 \frac{l_{d0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \varepsilon^{-\bar{\tau}(1-\rho)} \left(c_{d1} - \psi_0 \frac{l_{d1}^{1+\psi}}{1+\psi}\right)^{-\sigma} \tag{58}$$

The Euler equation for the choice of real assets for creditors at $t = 0$, h_{c1}, reads as follows:

$$[q_0 + X_1(h_{c1} - h_{c0}, h_{c0})] \left(c_{c0} - \psi_0 \frac{l_{c0}^{1+\psi}}{1+\psi}\right)^{-\sigma} =$$

$$\beta\left[\left(\bar{R} - 1\right) + q_1 + X_1(h_{c2} - h_{c1}, h_{c1}) - X_2(h_{c2} - h_{c1}, h_{c1})\right] \left(c_{c1} - \psi_0 \frac{l_{c1}^{1+\psi}}{1+\psi}\right)^{-\sigma}$$

After imposing that the economy is in steady state at $t = 1$ so that $q_1 = 1$ and $h_{c2} - h_{c1} = 0$, we obtain that

$$[q_0 + X_1(h_{c1} - h_{c0}, h_{c0})]\left(c_{c0} - \psi_0 \frac{l_{c0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \varepsilon^{-\bar{\tau}(1+\rho)}\left(c_{c1} - \psi_0 \frac{l_{c1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{59}$$

The analogous Euler equation for the $t = 0$ choice of real assets for debtors h_{d1} reads as follows

$$[q_0 + X_1(h_{d1} - h_{d0}, h_{d0})]\left(c_{d0} - \psi_0 \frac{l_{d0}^{1+\psi}}{1+\psi}\right)^{-\sigma} = \varepsilon^{-\bar{\tau}(1-\rho)}\left(c_{d1} - \psi_0 \frac{l_{d1}^{1+\psi}}{1+\psi}\right)^{-\sigma}, \tag{60}$$

By combining (57), (59), (58) and (60) one can see that $h_{j1} - h_{j0}$ $j = c, d$,can be different from zero only if $\rho \neq 0$—i.e. households have different beliefs about the future return of financial assets. ■

Lemma 6 *If*

$$B > \frac{\left|\varepsilon^{\frac{1}{\sigma}} - 1\right| \bar{N}}{2\bar{R}} + \frac{|\varepsilon^2 - 1| + \left(1 + \varepsilon^{\frac{1}{\sigma}}\right)\left(\bar{R} - 1\right)|\varepsilon - 1|}{4x_0\bar{R}} \tag{61}$$

fails the equilibrium of the model with the real asset features a credit crunch equilibrium where $B' = 0$ and output net of effort costs and adjustment costs $N_0 - X(\Delta)$ can be any value in the range $[N_0^{AH}, N_0^{BH}] \supset [N_0^A, N_0^B]$ where

$$N_0^{AH} = \bar{R}B + \min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}\left(\bar{N} - \bar{r}\Delta\right) - q_0\Delta$$

and

$$N_0^{BH} = -\bar{R}B + q_0\Delta + \max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}\left(\bar{N} + \bar{r}\Delta\right).$$

The price of the real asset q_0 and the amount of reallocation of the real assets Δ solve the following system of two equations

$$q_0 + X'(\Delta) = \max\{1, \varepsilon\} \tag{62}$$

$$q_0 - X'(\Delta) = \min\{1, \varepsilon\} \tag{63}$$

which imply that

$$\Delta = \frac{|\varepsilon - 1|}{2x_0} \quad \textit{and} \quad q_0 = \frac{1 + \varepsilon}{2} \tag{64}$$

If condition (61) holds, we have that financial markets are active $B' > 0$ and net output is equal to

$$N_0 - X(\Delta) = N_\varepsilon + \frac{\left(\bar{R}-1\right)\left[1 + \left(2\bar{R} - 1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]}{1 + \bar{R} + \left(\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}}|q_0 - 1|\Delta \tag{65}$$

where N_ε is output net of the effort costs of working in the baseline model with no real assets.

Proof of Lemma 6. The equilibrium beliefs of households still depend just of the end of period holdings of the financial assets a_{j1}, $j = c, d$ and in equilibrium are still fully characterized by Proposition 4. Total output (sum of output produced with labor and output of Lucas'trees) at time zero satisfies the aggregate resource constraint so that

$$\left(\bar{R} - 1\right) H + Y_0 = \frac{c_{c0} + c_{d0}}{2} + X(\Delta),$$

Again we have that clearing of the labor market implies that $\forall t\; Y_t = l_{jt}, \forall j = c, d$. Expected (with certainty) consumption at $t = 0$ and $t = 1$ of creditors is equal to

$$\begin{aligned} c_{c0} &= \left(\bar{R} - 1\right) H + Y_0 + \bar{R}\, a_{c0} - a_{c1} - q_0 \cdot (h_{c1} - h_{c0}) - X(h_{c1} - h_{c0}, 1) && (66)\\ c_{c1} &= \left(\bar{R} - 1\right) h_{c1} + \bar{Y} + \frac{\bar{R} a_{c1}}{\max\{1, \varepsilon\}} - a_{c2} - q_1 \cdot (h_{c2} - h_{c1}) - X(h_{c2} - h_{c1}, 1) \\ &= \left(\bar{R} - 1\right) h_{c1} + \bar{Y} + \frac{\left(\bar{R} - 1\right)}{\max\{1, \varepsilon\}} a_{c1} && (67) \end{aligned}$$

where ε denotes the monetary announcement and we used Proposition 4 to replace $\bar{\tau}$ and ρ. Expected (with certainty) consumption for debtors

$$\begin{aligned} c_{d0} &= \left(\bar{R} - 1\right) h_{d0} + Y_0 + \bar{R}\, a_{d0} - a_{d1} - q_0 \cdot (h_{d1} - h_{d0}) - X(h_{d1} - h_{d0}, 1) && (68)\\ c_{d1} &= \left(\bar{R} - 1\right) h_{d1} + \bar{Y} + \frac{\bar{R} a_{d1}}{\min\{1, \varepsilon\}} - a_{d2} - q_1 \cdot (h_{d2} - h_{d1}) - X(h_{d2} - h_{d1}, 1) \\ &= \left(\bar{R} - 1\right) h_{d1} + \bar{Y} + \left(\bar{R} - 1\right) \frac{a_{d1}}{\min\{1, \varepsilon\}} && (69) \end{aligned}$$

By combining (57) with (59) and after using the equilibrium beliefs of Proposition 4 we obtain the condition

$$q_0 + X'(h_{c1} - h_{c0}) = \max\{1, \varepsilon\} \tag{70}$$

Analogously by combining (58) with (60), and after using the equilibrium beliefs of Proposition 4 together with the condition for clearing in the market for the real asset we obtain the condition

$$q_0 - X'(h_{c1} - h_{c0}) = \min\{1, \varepsilon\}. \tag{71}$$

By subtracting side by side (70) and (71) we obtain

$$2X'(h_{c1} - h_{c0}) = |\varepsilon - 1|$$

which implies that

$$\Delta = h_{c1} - h_{c0}. \tag{72}$$

This says that independently of whether the announcement is inflationary ($\varepsilon > 1$) or deflationary ($\varepsilon < 1$), the real asset is reallocated from debtors to creditors. This is because the expected return on the financial asset is always smaller form creditors than for debtors, so debtors are the least likely to invest in the real asset (whose return is constant and unaffected by monetary policy). By imposing our functional form of the adjustment cost function X and then solving the system (62) and (63), we obtain that

$$\begin{aligned}
\Delta &= \frac{|\varepsilon - 1|}{2x_0} \\
q_0 &= \frac{1+\varepsilon}{2} \\
q_0\Delta &= \frac{|\varepsilon^2 - 1|}{4x_0} \\
X(\Delta) &= \frac{(\varepsilon - 1)^2}{4x_0} \\
q_0\Delta + X(\Delta) &= \frac{|\varepsilon^2 - 1| + (\varepsilon - 1)^2}{4x_0} \\
q_0\Delta - X(\Delta) &= \frac{|\varepsilon^2 - 1| - (\varepsilon - 1)^2}{4x_0} > 0
\end{aligned}$$

Notice that $q_0\Delta + X(\Delta)$ is the total financial cost incurred by creditors at $t = 0$ in order to buy additional units of the real asset, while $q_0\Delta - X(\Delta)$ is the total financial income received by debtors at $t = 0$ to sell some units of the real asset they owned. Notice that these quantities are both zero if x_0 is equal to ∞, which coincides with the baseline model. Quadratic adjustment costs have the property that the x_0 parameter does not affect equilibrium prices but just the amount of reallocation Δ. Notice that the expression for equilibrium prices would also hold if adjustment costs are homogeneous of degree one. ■

Now we can solve for the equilibrium in the financial market. We use the conditions for financial market clearing at $t = -1$, $a_{c0} = -a_{d0} = B$, and at $t = 0$, $a_{c1} = -a_{d1} = B'$ together with (72) and (57) and (58) evaluated at the equilibrium beliefs of Proposition 4. We then obtain

$$\frac{\bar{N} + \frac{\bar{r}}{\max\{1,\varepsilon\}} B' + \bar{r}\Delta}{\widehat{N}_0 + \bar{R}B - B' - q_0\Delta} = \frac{1}{\max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}} \tag{73}$$

$$\frac{\bar{N} - \frac{\bar{r}}{\min\{1,\varepsilon\}} B' - \bar{r}\Delta}{\widehat{N}_0 - \bar{R}B + B' + q_0\Delta} = \frac{1}{\min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\}}, \tag{74}$$

where $\bar{N} \equiv N^H(\bar{Y}, H)$ and $\widehat{N}_0 \equiv N^H(Y_0, H) - X(\Delta)$, representing output net of the effort cost of working.

We now solve for N_0. From (74) we obtain

$$\left[\bar{N} - \frac{\overline{r}}{\min\{1,\varepsilon\}} B' - \overline{r}\Delta\right] \min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} = \widehat{N}_0 - \bar{R}B + B' + q_0\Delta$$

so that

$$\min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} \left(\bar{N} - \overline{r}\Delta\right) + \bar{R}B - q_0\Delta - \widehat{N}_0 = \left[1 + \frac{\overline{r}}{\min\left\{1, \varepsilon^{1-\frac{1}{\sigma}}\right\}}\right] B'$$

so that

$$B' = \frac{\bar{R}B + \min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} \left(\bar{N} - \overline{r}\Delta\right) - q_0\Delta - \widehat{N}_0}{1 + \frac{\overline{r}}{\min\left\{1, \varepsilon^{1-\frac{1}{\sigma}}\right\}}} \tag{75}$$

From (73) we have

$$B' = \frac{\bar{R}B - \max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} \left(\bar{N} + \overline{r}\Delta\right) - q_0\Delta + \widehat{N}_0}{1 + \frac{\overline{r}}{\max\left\{1, \varepsilon^{1-\frac{1}{\sigma}}\right\}}}. \tag{76}$$

The equilibrium is interior if the intercept on the y- axis of the schedule implicitly defined by (74) in the net output space $\widehat{N}_0 \equiv N_0 - X(\Delta)$ and end of period zero imbalances B' is above the intercept on the y-axis of the (73) schedule. Net output is defined as net of adjustment costs, $\widehat{N}_0 \equiv N_0 - X(\Delta)$.
Credit crunch equilibrium The intercept of (SA) on the y-axis is given by

$$N_0^{AH} = \bar{R}B + \min\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} \left(\bar{N} - \overline{r}\Delta\right) - q_0\Delta$$

The intercept of (73) on the y-axis is given

$$N_0^{BH} = -\bar{R}B + q_0\Delta + \max\left\{1, \varepsilon^{\frac{1}{\sigma}}\right\} \left(\bar{N} + \overline{r}\Delta\right)$$

The condition $N_0^{AH} \geq N_0^{BH}$ is equivalent to

$$B > \frac{\left|\varepsilon^{\frac{1}{\sigma}} - 1\right| \bar{N}}{2\bar{R}} + \frac{q_0\Delta}{\bar{R}} + \frac{\left(1 + \varepsilon^{\frac{1}{\sigma}}\right)}{2\bar{R}} \left(\bar{R} - 1\right)\Delta \tag{77}$$

After using our functional for adjustment costs, which imply that $q_0\Delta = \frac{|\varepsilon^2 - 1|}{4x_0}$ and $\Delta = \frac{|\varepsilon - 1|}{2x_0}$, we obtain (61) in the main text. If (61) fails we have a credit crunch equilibrium. In a credit crunch equilibrium $B' = 0$ and any net output level in the range $[N_0^{AH}, N_0^{BH}]$ can be sustained as an equilibrium. But here we considered just the possibility that a credit crunch equilibrium exists with no effects on the amount of reallocation. But in a credit crunch equilibrium beliefs are undetermined and if we assume that creditors and debtors have the same beliefs we would have that $\Delta = 0$ from Lemma **??**. Under equal beliefs we can sustain any net output level in the range $[N_0^A, N_0^B]$ where N_0^A and N_0^B they correspond to the thresholds for a credit crunch equilibrium to arise in the baseline model, as given in (21) and (22). We generally have

$$N_0^{AH} < N_0^A \qquad \text{and} \qquad N_0^{BH} > N_0^B \tag{78}$$

which implies that $[N_0^A, N_0^B] \subseteq [N_0^{AH}, N_0^{BH}]$. This says that credit crunch equilibrium are more likely when households can trade in real assets than in the baseline model with just one financial asset. The result in (78) also guarantees that a credit crunch equilibrium and an equilibrium with $B' > 0$ can never coexist. The choice of beliefs can determine the equilibrium amount of reallocation Δ and the equilibrium price of the real asset q_0. $[N_0^A, N_0^B]$ represent the set of net output levels $\widehat{N}_0$ that can be sustained when in a credit crunch equilibrium all households share the same beliefs, $[N_0^{AH}, N_0^{BH}] \backslash [N_0^A, N_0^B]$ represent the net output level $\widehat{N}_0$ that can be sustained when we allow creditors and debtors to have different beliefs in a credit crunch equilibrium.

Start by noticing that in the baseline model with just one financial asset independently of whether the announcement is inflationary $\varepsilon > 1$ or deflationary $\varepsilon < 1$, and the equilibrium does not feature a credit crunch we have

$$N_\varepsilon = \omega_\varepsilon \bar{N} - \zeta_\varepsilon \left| 1 - \varepsilon^{\frac{1}{\sigma}-1} \right| B, \tag{79}$$

where

$$\omega_\varepsilon = \frac{1 + (\bar{R} - 1)\,\varepsilon^{\frac{1}{\sigma}-1} + \bar{R}\varepsilon^{\frac{1}{\sigma}}}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma}-1}}$$
$$\zeta_\varepsilon \equiv \frac{\bar{R}\,(\bar{R} - 1)}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma}-1}}.$$

Net output after a deflationary announcement $\varepsilon < 1$ *and* $B' > 0$. Let's now assume that condition (61) holds so that in equilibrium $B' > 0$ and consider the case of a deflationary announcement $\varepsilon < 1$. Since the equilibrium beliefs are still given by Proposition 4, from (76) we have that

$$B' = B + \frac{N_0 - X(\Delta) - q_0\Delta - \bar{N} - \left(\bar{R} - 1\right)\Delta}{\bar{R}}$$

which can be substituted into (75) to obtain after some algebra

$$\widehat{N}_0 \equiv N_0 - X(\Delta) = N_\varepsilon + \frac{\left(\bar{R} - 1\right)\left[\bar{R}\varepsilon^{\frac{1}{\sigma}-1}(1 - \epsilon) - \left(\varepsilon^{\frac{1}{\sigma}-1} - 1\right)(1 - q_0)\right]}{1 + \bar{R} + (\bar{R} - 1)\varepsilon^{\frac{1}{\sigma}-1}}\Delta$$

where N_ε is net output in the baseline model as given in (79). Under our functional form for adjustment costs we have $1-q_0 = \frac{1-\varepsilon}{2}$, which can be substituted into the above equation to obtain

$$\widehat{N}_0 = N_\varepsilon + \frac{\left(\bar{R}-1\right)\left[1+\left(2\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]}{1+\bar{R}+(\bar{R}-1)\varepsilon^{\frac{1}{\sigma}-1}}(1-q_0)\Delta \tag{80}$$

Net output after an inflationary announcement $\varepsilon > 1$ If we assume that condition (61) holds (so that in equilibrium $B' > 0$) and we consider the case of a inflationary announcement $\varepsilon > 1$, we can use Proposition 4 to rewrite condition (75) as follows

$$B' = \frac{\bar{R}B + \left(\bar{N} - \bar{r}\Delta\right) - q_0\Delta + X(\Delta) - N_0}{\bar{R}}$$

which can be used to replace B' in (76). After using again Proposition 4 with $B' > 0$ we obtain

$$\begin{aligned} &\left[1+\left(\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]B + \left[1+\left(\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]\frac{\bar{N}-\left(\bar{R}-1\right)\Delta - q_0\Delta - \widehat{N}_0}{\bar{R}} \\ = \; &\widehat{N}_0 + \bar{R}B - q_0\Delta - \varepsilon^{\frac{1}{\sigma}}\left(\bar{N}+\bar{r}\Delta\right) \end{aligned}$$

which, after some algebra, allows us to conclude that

$$\widehat{N}_0 = N_\varepsilon + \frac{\left(\bar{R}-1\right)\left[1+\left(2\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}\right]}{1+\bar{R}+\left(\bar{R}-1\right)\varepsilon^{\frac{1}{\sigma}-1}}\left(q_0-1\right)\Delta \tag{81}$$

where N_ε is again given in (79).

Conditions (80) and (81) prove that the expression in (65) holds true.

A.3 The model of Section 3 with alternative modeling of ambiguity aversion

Here we assume that households have multiplier preferences with respect to ambiguity as in Hansen and Sargent (2001, 2008).

Households have a reference probability distribution $\hat{G}(\Pi_1^*)$ but consider the possibility that $\hat{G}$ may not be the appropriate law that governs the inflation target implemented by the central bank, and therefore consider alternative models G. The relative likelihood of these alternative models is measured by the relative entropy, so that the agents act on a probability distribution that is chosen as

$$\min_G \left[\int_{\underline{\Pi}_1^*}^{\bar{\Pi}_1^*} v\left(\frac{a_1}{\Pi_1^*}\right) dG(\Pi_1^*) + \lambda\, R(G|\hat{G})\right]$$

where

$$R(G|\hat{G}) = \int_{\underline{\Pi}_1^*}^{\bar{\Pi}_1^*} \log\frac{dG(\Pi_1^*)}{d\hat{G}(\Pi_1^*)} dG(\Pi_1^*)$$

is the relative entropy of G with respect to $\hat{G}$. Suppose $\hat{G}$ has density $\hat{g}$. Rewrite the problem as

$$\min_{G}\left[\int_{\underline{\Pi}_1^*}^{\bar{\Pi}_1^*} v\left(\frac{a_1}{\Pi_1^*}\right) g(\Pi_1^*)d\Pi_1^* + \lambda \int_{\underline{\Pi}_1^*}^{\bar{\Pi}_1^*} \log\left(\frac{g(\Pi_1^*)}{\hat{g}(\Pi_1^*)}\right) g(\Pi_1^*)d\Pi_1^*\right]$$

The first order condition implies:

$$v\left(\frac{a_1}{\Pi_1^*}\right) + \lambda \log\left(\frac{g(\Pi_1^*)}{\hat{g}(\Pi_1^*)}\right) + \lambda = 0$$

implying

$$g(\Pi_1^*) = \hat{g}(\Pi_1^*)\left[\exp\left(-v\left(\frac{a_1}{\Pi_1^*}\right) - \lambda\right)\right]^{\frac{1}{\lambda}} = \hat{g}(\Pi_1^*) \exp\left(-\frac{v\left(\frac{a_1}{\Pi_1^*}\right)}{\lambda}\right) \exp(-1)$$

Finally we use $\int g(\Pi^*)d\Pi^* = \int g(\Pi^*)d\Pi^* \hat{g}(\Pi_1^*) \exp\left(-\frac{v\left(\frac{a_1}{\Pi_1^*}\right)}{\lambda}\right) \exp(-1) = 1$ and obtain

$$g(\Pi_1^*) = \frac{\hat{g}(\Pi_1^*) \exp\left(-\frac{v\left(\frac{a_1}{\Pi_1^*}\right)}{\lambda}\right)}{\int_{\underline{\Pi}^*}^{\bar{\Pi}^*} \hat{g}(\Pi_1^*) \exp\left(-\frac{v\left(\frac{a_1}{\Pi_1^*}\right)}{\lambda}\right) d\Pi_1^*}$$

Notice that similar equations would apply in the case that the reference distribution for Π_1^* is a discrete one. In this case, we assume that $g(\Pi_1^*) = 0$ for all Π_1^* at which $\hat{g}(\Pi_1^*) = 0$, i.e. if the reference probability assign zero likelihood to an event then also the subjective probability will do so.

Consider the following example. Suppose that $\hat{g}(\Pi_1^*)$ is such that the household considers possible only the case where the central bank does not implement and the case where it fully implements, i.e. $\hat{g}(1) = 1/2$ and $\hat{g}(\varepsilon) = 1/2$. The problem of the household becomes:

$$V(a_{j0}) = \max_{c,l,a'}\left\{U(c,l) + \beta\, \frac{\frac{1}{2}\exp\left(-\frac{\bar{V}(a_1)}{\lambda}\right)\bar{V}(a') + \frac{1}{2}\exp\left(-\frac{\bar{V}\left(\frac{a_1}{\varepsilon}\right)}{\lambda}\right)\bar{V}\left(\frac{a_1}{\varepsilon}\right)}{\frac{1}{2}\exp\left(-\frac{\bar{V}(a_1)}{\lambda}\right) + \frac{1}{2}\exp\left(-\frac{\bar{V}\left(\frac{a_1}{\varepsilon}\right)}{\lambda}\right)}\right\}$$

$$\text{s.t.} \quad c + a' \leq w_0\, l + \bar{R}\, a_{j0} + \lambda_0,$$

where the continuation utility is

$$\bar{V}(s) = \frac{\left[\bar{N} + (\bar{R}-1)s\right]^{1-\sigma}}{(1-\sigma)(1-\beta)},$$

Let

$$h(a_1) \equiv \exp\left(\frac{\bar{V}\left(\frac{a_1}{\varepsilon}\right) - \bar{V}(a_1)}{\lambda}\right)$$

then the first order condition for the optimal choice of a_1 is

$$U_1'(c,l) = \beta\,\frac{h(a_1)\,\bar{V}'(a_1) + \frac{1}{\varepsilon}\,\bar{V}'\left(\frac{a_1}{\varepsilon}\right)}{h(a_1)+1} + \beta\,\frac{h'(a_1)}{h(a_1)+1}\left[1 - \frac{h(a_1)\,\bar{V}(a_1) + \bar{V}\left(\frac{a_1}{\varepsilon}\right)}{h(a_1)+1}\right]$$

We notice that if $\varepsilon > 1$ (< 1) then $h(a_1)$ is decreasing (increasing) in a_1 and that $\bar{V}'(a_1) - \frac{1}{\varepsilon}\bar{V}'\left(\frac{a_1}{\varepsilon}\right)$ is increasing (decreasing) in a_1.

The set of equations that characterize the equilibrium of the economy in this case are given by

$$\begin{aligned}
U_1'(c_c, Y_0) &= \beta\,\frac{h(B')\,\bar{V}'(B') + \frac{1}{\varepsilon}\,\bar{V}'\left(\frac{B'}{\varepsilon}\right)}{h(B')+1} + \beta\,\frac{h'(B')}{h(B')+1}\left[1 - \frac{h(B')\,\bar{V}(B') + \bar{V}\left(\frac{B'}{\varepsilon}\right)}{h(B')+1}\right]\\
U_1'(c_d, Y_0) &= \beta\,\frac{h(-B')\,\bar{V}'(-B') + \frac{1}{\varepsilon}\,\bar{V}'\left(\frac{-B'}{\varepsilon}\right)}{h(-B')+1} + \beta\,\frac{h'(-B')}{h(-B')+1}\left[1 - \frac{h(-B')\,\bar{V}(-B') + \bar{V}\left(\frac{-B'}{\varepsilon}\right)}{h(-B')+1}\right]\\
c_c &= Y_0 + \bar{R}\,B - B'\\
c_d &= Y_0 - \bar{R}\,B + B'\\
h(x) &= \exp\left(\frac{\bar{V}\left(\frac{x}{\varepsilon}\right) - \bar{V}(x)}{\lambda}\right)\\
h'(x) &= -h(x)\left[\bar{V}'(x) - \frac{1}{\varepsilon}\,\bar{V}'\left(\frac{x}{\varepsilon}\right)\right]\\
\bar{V}(x) &= \frac{\left[\bar{N} + (\bar{R}-1)x\right]^{1-\sigma}}{(1-\sigma)(1-\beta)},\\
\bar{V}'(x) &= (1-\sigma)\,\bar{V}(x)\,\frac{\bar{R}-1}{\bar{N} + (\bar{R}-1)x},\\
\bar{N} &= \bar{Y} - \psi_0\,\frac{\bar{Y}^{1+\psi}}{1+\psi}
\end{aligned}$$

B Data appendix

We describe the sources of our data for realized and expected inflation in the Italian provinces, and in the euro area, as well as the net financial assets of European households.

B.1 Italian data

Our Italian data come from ISTAT's Survey of Inflation Expectations conducted by the Bank of Italy and Sole24Ore (Italy's main daily business paper), and from the Bank of Italy's Survey of Household Income and Wealth.

Realized inflation at province level is taken directly from ISTAT's "I.Stat" online archive. We use the general price index, pgen in the ISTAT database. Realized inflation in the province corresponds to the yearly log-difference of pgen in the province. We take yearly log-differences because the ECB monitors price stability on the basis of the annual rate of change in HICP and because of the working of the inflation expectations question (see below).

Expected inflation measures 2 quarters ahead expected inflation, averaging the reported estimates of all observations in the province in the Survey of Inflation Expectations. The disaggregated province level data are confidential data kindly made available to us by the Bank of the Italy. The Survey has been conducted quarterly since 1999, in March, June, September and December. The sample comprises about 800 companies, operating in all industries including construction. Individuals are asked to predict the price inflation 6 months ahead, answering the following question: "[If the survey is conducted in June 2013] What do you think consumer price inflation in Italy, measured by the 12-month change in the Harmonized Index of Consumer Prices (HICP), will be in December 2013?". Note that the individuals in the survey are all asked to predict the evolution of the same index (HICP at the national level) . In practice, therefore we are assuming that the replies of respondents in the survey in that province reflect the average beliefs of agents in the province.

Net Financial Assets (NFA) Our data on the Italian households' NFA come from the Survey of Household Income and Wealth (SHIW), administered by the Bank of Italy on a representative sample of Italian households. The survey, which is biannual, collects detailed data on households' finances. Each wave surveys about 8,000 households, which, applying weights provided by SHIW (mnemonic Pesofit in SHIW), are fully representative of the Italian resident population. To increase sample size, we use both the 2010 and the 2012 waves. NFA is calculated as the difference between the sum of households' holdings of postal deposits, saving certificates and CDs (mnemonic shiwaf1 in SHIW), government securities (mnemonic shiwaf2) and other securities (mnemonic shiwaf3) minus the sum of their financial liabilities to banks and other financial companies (mnemonic shiwpf1), trade debt (mnemonic shiwpf2) and liabilities to other households (mnemonic shiwpf3).

Creditor households are those with positive NFA (see the construction of the variable NFA for details).

Fraction of creditor households For each province we calculate the pre-announcement fraction of creditor households, based on the 2010 and 2012 waves of SHIW, weighting each household according to the weights provided by SHIW (mnemonic Pesofit).

Inflation expectation bias In each province i and quarter t, we calculate the difference between expected inflation and future realized inflation, which corresponds to equation (82) in the main text.

Table A1: Descriptive statistics

VARIABLES	(1) mean	(2) sd	(3) N	(4) min	(5) max
A) Italian Micro data					
Pre-announcement fraction of creditor households	0.66	0.13	1078	0.32	0.94
Pre-announcement fraction of creditor households, divided by SD	-0.13	1.00	1078	-2.76	2.00
Inflation rate in province π_{it}	1.77	1.24	1078	-0.47	4.76
Two quarters ahead expected inflation, $E_{it}[\pi_{it+2}]$	2.02	1.23	1078	-10	8.72
Two quarters ahead realized inflation, π_{it+2}	1.15	1.16	1078	-9.62	4.53
Inflation expectation bias, $\hat{\pi}_{it}$	0.86	0.74	1078	-3.61	6.79
Year	2012.80	0.75	1082	2012	2014
B) Euro area data					
Net per capita financial assets	1.91	1.57	100	-0.42	4.67
Net per capita financial assets, divided by SD	1.22	1	100	-0.27	2.98
Fraction of households who think inflation will increase in next 12 months	15.99	6.08	100	6	38.10
Inflation rate in country	1.65	0.77	100	-0.05	3.09
Change in Country Inflation rate	-0.34	0.60	100	-2.39	1.18
Year	2012.80	0.75	100	2012	2014

Notes: Quarterly data over the sample period 2012:I-2014:II. Realized inflation comes from ISTAT. Data on expected inflation are based on confidential data from the Bank of Italy-Sole 24Ore survey on expectations. The Net Financial Asset position of households is calculated using the 2010 and 2012 waves of the Survey of Household Income of Wealth (SHIW). Euro Area data come from ECB, Joint Harmonized Programme of Business and Consumer Surveys by European Commission and Eurosystem Household Finance and Consumption Survey (HFCS).

B.2 Euro Area Data: realized and expected inflation

The data are for the Euro 11 countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain.

Core Inflation is the yearly log differences in the Harmonised Index of Consumer Prices (HICP), net of energy and unprocessed food, multiplied by 100, taken from the Eurostat data warehouse available at "http://ec.europa.eu/eurostat/".

Fraction of households who think inflation will increase in next 12 months come from the European Commission's Business and Consumer Surveys. The key advantage of the Consumer Survey is that it directly asks households for their expectations about future inflation, which distinguishes it sharply from the commonly used Survey of Professional Forecasters. Sample size varies with country. Price expectations are derived from the question: "By comparison with the past 12 months, how do you expect that consumer prices will develop in the next 12 months? They will (i) increase more rapidly; (ii) increase at the same rate; (iii) increase at a slower rate; (iv) stay about the same; (v) fall. The fraction of households who think inflation will increase in next 12 months is tteh fraction of households who selecting option i). The series are seasonally adjusted by the Commission.

B.3 European households' Net Financial Assets in HFCS

The Eurosystem Household Finance and Consumption Survey (HFCS) collects fully harmonized data on households' portfolio asset allocation of households and consumption expenditures in the Euro-11 countries (except Ireland). Wealthy individuals are over-sampled for better characterization of the right tail of the income and wealth distribution. Within each country, the sum of the estimation weights equals the total number of households, so that the sum of weights in the entire dataset equals the total number of households in the ten countries of Euro 11 we consider. The structure of the HFCS resembles that of the US Survey of Consumer Finances. To account for measurement error and missing observations, HFCS reports five separate imputation replicates (implicates) for each record. All statistics are calculated by the procedure recommended by HFCS: for each implicate we calculate the desired statistic using HFCS weights (mnemonic hw0010) and then average across the five implicates (mnemonic im0100). The survey was carried out in 2010 except in Finland and the Netherlands, where it was done in 2009, and in Spain (2008). All statistics are at constant 2010 prices.

Net Financial Assets (NFA) is calculated as the the difference between total financial assets and total financial liabilities. Financial assets are (i) deposits (mnemonic da2101); (ii) mutual funds (mnemonic da2102); (iii) bonds (mnemonic da2103); (iv) non self-employment private business (mnemonic da2104); (v) value of self-employment business (mnemonic da1140); (vi) shares of publicly traded companies (mnemonic ds2105); (vii) managed accounts (mnemonic da2106); (viii) money owed to households (mnemonic da2107); (ix) other assets (mnemonic da2108); and (x) voluntary pensions plus whole life insurance (mnemonic da2109). Financial liabilities are the sum of (i) outstanding balance of mortgages on household's main residence (mnemonic dl1110); (ii) outstanding balance of mortgages on other properties (mnemonic dl1120); and (iii) outstanding balance of other non mortgage debt (mnemonic dl1200).

Net Financial Assets net of public debt is obtained by subtracting the country's per household government debt from the household's NFA. The household's country of residence is obtained from mnemonic sa0100. Per household government debt is the country-specific level of net government debt per capita as reported in Table 1 of Adam and Zhu (2015) multiplied by the average number of household members older than 16 as obtained by HFCS (mnemonic dh0006).

Consumption expenditures is the sum of the expenditures during the last 12 months on food and beverages at home (mnemonic hi0100) and on food and beverages outside the home (mnemonic hi0200).

Average labor income in the euro area is the average of the employee income of all household members (mnemonic di1100) for all households whose head is aged 20-65 (mnemonic ra0300). The resulting average labor income is EUR 21,631.

B.4 Robustness with Euro Area data

To check robustness about the increase in disagreement among European households about expected future inflation after forward guidance we also analyzed country level evidence for the Euro 11 Area. For countries we do not have information on expected inflation but just on the fraction of households who think that inflation will increase in the next year relative to the past year, which we denote by $P\left(E_{it}[\pi_{it+4}] - \pi_{it}\right)$. Then we calculate

$$\hat{\pi}_{it} \equiv P\left(E_{it}[\pi_{it+4}] - \pi_{it}\right) - \vartheta\left(\pi_{it+4} - \pi_{it}\right) \tag{82}$$

where ϑ will be estimated. To evaluate whether, in response to forward guidance, the inflation expectation bias has increased more for creditor households than for debtor households, we run the the same Difference-in-Differences regression as in (43), but where F_i is now equal to the (standardized) average per capita Net Financial Asset of households in the country. The controls X_{it} includes a full set of time and country dummies and also the realized future inflation which allows to estimate ϑ in 82. The results from estimating (43) with the Euro 11 data are reported in Table A2. Column 1 reports the results discussed in the main text, column 2 reports the result with the sample of countries. The evidence indicates that the inflation expectations have become more correlated with the NFA of households and this conclusion is confirmed when focusing on the sample of Euro 11 countries.

Table A2: FG Effects on expected inflation bias

VARIABLES	**Micro Evidence** $\hat{\pi}^c_{it}$	**EURO 11** $\hat{\pi}^p_{it}$
Announcement-dummy $\times$ F_i (coefficient ϕ)	.10***	2.1***
	.04	.71
Effect of financial position F_i (coefficient $\overline{\phi}$)	.02	10.39***
	.02	.95
Future changes in inflation $\pi_{it+4} - \pi_{it}$, γ		2.33***
		.75
R-squared	.35	.97
No. of observations	1,078	100
No. of i units	110	10

Notes: Results from regression (43). All regressions include year and individual fixed effect. The dependent variable is $\hat{\pi}^j_{it}$, in (43) and (82). The sample period is 2012:I-2014:II. F_i is the (standardized) pre-announcement fraction of households with positive NFA in the province in column (1) or the (standardized) pre-announcement average value of households' NFA in the country in column (2). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.10.

C Computational details

We solve the extended model of Section 4 in four steps. In the first step we guess the nominal interest rate at $t = T$, $R_T = \hat{R}_T$. In the second step, we take the guess for R_T as given and construct the function $\bar{V}(a_{xT}, \Pi^*_T)$ in (**??**) by solving for the equilibrium of the economy at $t \geq T$, for different values of Π^*_T. In the third step we take the function

$\bar{V}(a_{xT}, \Pi^*_T)$ as given from step 2 and solve for the equilibrium of the economy at $t < T$. This yields a new value for R_T equal to $\hat{R}'_T$. In the fourth step we check convergence. We describe the four steps below.

Step 1 We guess a value for R_T, say $\hat{R}$.

Step 2 Given $R_T = \hat{R}$, we solve the equilibrium of the model at $t \geq T$ under two scenarios: $\Pi^*_T = \varepsilon$ and $\Pi^*_T = 1$. Notice that Π^*_T fully determines Π^*_t $\forall t \geq T$. We index equilibrium quantities by the superscript 1 if $\Pi^*_T = \varepsilon$; by the superscript 0 if $\Pi^*_T = 1$. Then, $\forall m = 0, 1$, we guess a path of output, $\{\hat{Y}^m_t\}_{t\geq T}$. Notice that when $R_T = \bar{R}$, $Y^0_t = \bar{Y}$ $\forall t \geq T$ (see Proposition 7). Given an output path, (11) together with the labor market clearing condition yields a path for wages, $\{\hat{w}^m_t\}_{t\geq T}$. Given output and wages, (3) and (38) jointly determine the path of inflation $\{\hat{\Pi}^m_t\}_{t\geq T}$ and nominal interest rates $\{\hat{R}^m_t\}_{t\geq T}$, where interest rates satisfy $\hat{r}^m_t = \hat{R}^m_t / \hat{\Pi}^m_t$. The path of dividends $\{\hat{D}^m_t\}_{t\geq T}$ is obtained using (39). Then we obtain aggregate consumption $\{\hat{C}^m_t\}_{t\geq T}$ from (36). Given the path of inflation $\{\hat{\Pi}^m_t\}_{t\geq T}$ and aggregate consumption $\{\hat{C}^m_t\}_{t\geq T}$, we apply (40) to obtain a new sequence of output, denoted by $\{Y^m_t\}_{t\geq T}$. If $\max_{t\geq T} |Y^m_t - \hat{Y}^m_t| < |\varepsilon - 1| \times 10^{-5}$ we stop, and the initial guess for the output sequence $\{\hat{Y}^m_t\}_{t\geq T}$ is verified; otherwise we use $\{Y^m_t\}_{t\geq T}$ to update the guess for $\{\hat{Y}^m_t\}_{t\geq T}$ and reiterate until convergence. After achieving convergence for $m = 0, 1$, we construct the function $\bar{V}(a_{xT}, \Pi^*_T)$ in (**??**), find a^* such that $\bar{V}(a^*, \varepsilon) = \bar{V}(a^*, 1)$, and then go to step 3.

Step 3 For all $t < T$, we conjecture a path of output, $\{\hat{Y}_t\}_{t<T}$. Given $\{\hat{Y}_t\}_{t<T}$, (11) together with the labor market clearing condition yields a path of wages, $\{\hat{w}_t\}_{t<T}$. Let $\{\tilde{w}_{xt}\}_{t\geq 0}$ and $\{\tilde{Y}_{xt}\}_{t\geq 0}$ denote household x's beliefs about the path of wages and output, respectively. Since households share the same beliefs about all variables $\forall t < T$, we have that $\{\tilde{w}_{xt}\}_{t\geq 0} = \{\hat{w}_0, \hat{w}_1, ..\hat{w}_{T-1}, \hat{w}^1_T, \hat{w}^1_{T+1}, ...\}$ if $a_{xT} > a^*$ and $\{\tilde{w}_{xt}\}_{t\geq 0} = \{\hat{w}_0, \hat{w}_1, ..\hat{w}_{T-1}, \hat{w}^0_T, \hat{w}^0_{T+1}, ...\}$ if $a_{xT} < a^*$. Given $\{\tilde{w}_{xt}\}_{t\geq 0}$, (3) and (38) determine household x's beliefs about the path of the interest rate $\{\tilde{r}_{xt}\}_{t\geq 0}$ and inflation $\{\tilde{\pi}_{xt}\}_{t\geq 0}$. There are then three cases to consider: (i) household x believes the announcement, which requires $a_{xT} > a^*$; (ii) she does not believe the announcement, which requires $a_{xT} < a^*$; (iii) she has degenerate beliefs, which requires $a_{xT} = a^*$. In cases (i) and (ii), household x's consumption can be obtained by integrating forward the Euler condition in (35) to yield

$$c_{xt} = \frac{\psi_0}{1+\psi} Y_t^{1+\psi} + \left(\sum_{s=0}^{\infty} \beta^{\frac{s}{\sigma}} \tilde{q}_{xt,t+s}^{1-\frac{1}{\sigma}} \right)^{-1} \left(a_{xt}\, r_t + \frac{\psi_0 \psi}{1+\psi} \sum_{s=0}^{\infty} \tilde{q}_{xt,t+s}\, \tilde{Y}_{xt+s}^{1+\psi} \right), \tag{83}$$

where $\tilde{q}_{xt,t+s} = \left(\prod_{n=1}^{s} \tilde{r}_{xt+n}\right)^{-1}$. We use (83) to calculate c_{xt} both under the assumption that household x believes the announcement, $m = 1$, and under the assumption that it does not, $m = 0$. Applying the household's budget constraint, we obtain an associated value of a_{xT} denoted by a^m_{xT}, $\forall m = 0, 1$. If $a^1_{xT} > a^*$, then household x believes the announcement (case (i)); if $a^0_{xT} < a^*$, it does not believe the announcement (case (ii)). Notice that $a^1_{xT} > a^*$ and $a^0_{xT} < a^*$ cannot both hold. If neither $a^1_{xT} > a^*$ nor $a^0_{xT} < a^*$ is verified, we have case (iii), so $a_{xT} = a^*$, which can be used together with (35) to determine the path of consumption of household x $\forall t < T$. Once we have $\{c_{xt}\}_{t<T}$ $\forall x \in [0,1]$, we calculate $C_t = \int_0^1 c_{xt}\, dx$ and use (40) together with (38) to obtain a new sequence of output $\forall t < T-1$, which is denoted by $\{\hat{Y}_t'\}_{t<T}$. If $\max_{t<T} |\hat{Y}_t' - \hat{Y}_t| < |\varepsilon - 1| \times 10^{-5}$ we stop, use (3) to calculate R_T, set $\hat{R}_T' = R_T$, and go to step 4; otherwise we use $\{\hat{Y}_t'\}_{t<T}$ to update the guess for $\{\hat{Y}_t\}_{t<T}$ and iterate until convergence.

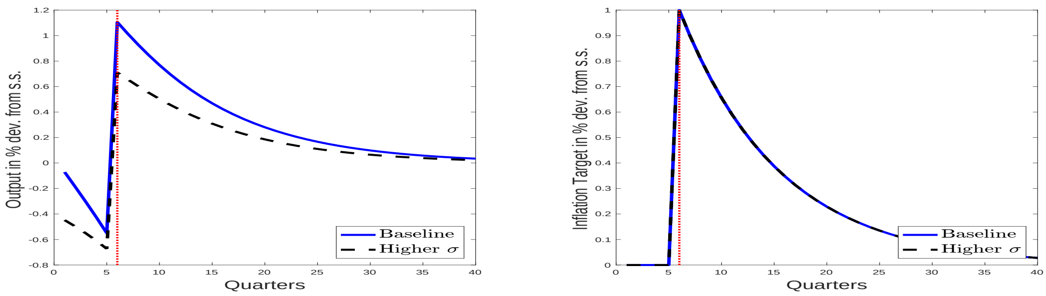
Step 4 If $|\hat{R}_T' - \hat{R}_T| > 10^{-4}\, |\varepsilon - 1|$ we go back to step 1 and use $\hat{R}_T'$ to update our guess for R_T to a new $\hat{R}_T$; otherwise the algorithm finally converges and we stop.

D Further robustness checks

D.1 Intertemporal elasticity of substitution

In this Section we increase σ from 2 to 2.5, corresponding to a drop in the elasticity of intertemporal substitution from 0.5 to 0.4. Figure A1 shows that a lower EIS makes forward guidance less expansionary: relative to the baseline specification, output falls more $\forall t < T$, and increases less $\forall t \geq T$.

Figure A1: Response to the forward guidance announcement with $\sigma = 2.5$



(a) Output Y (b) Inflation Target Π^*

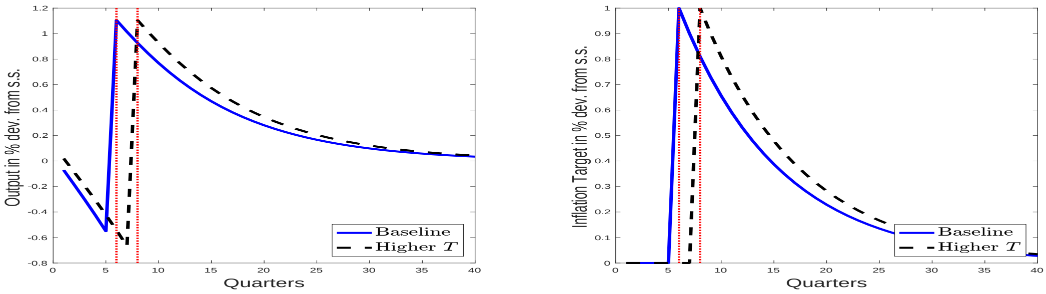
D.2 Horizon of the announcement

In this section we consider an announcement at a time horizon of 2 years, $T = 8$. Figure A2 shows that output falls less on impact than in the baseline specification ($T = 6$). This is because the expected cumulative decline in the interest rate before T is mechanically larger when T is farther ahead. Thus, the substitution effect on consumption is relatively stronger, which makes output increase more (or decrease less) on impact. As we approach T, untrusting households expect the real rate to go back to steady state sooner and reduce their consumption more, decreasing output relative to the baseline specification.

D.3 Taylor rule at $t < T$

In this section we assume that the nominal interest rate follows the Taylor rule in (3) also in the interim period period up to T. We reestimate the path of the inflation target to

Figure A2: Response to the forward guidance announcement with $T = 8$

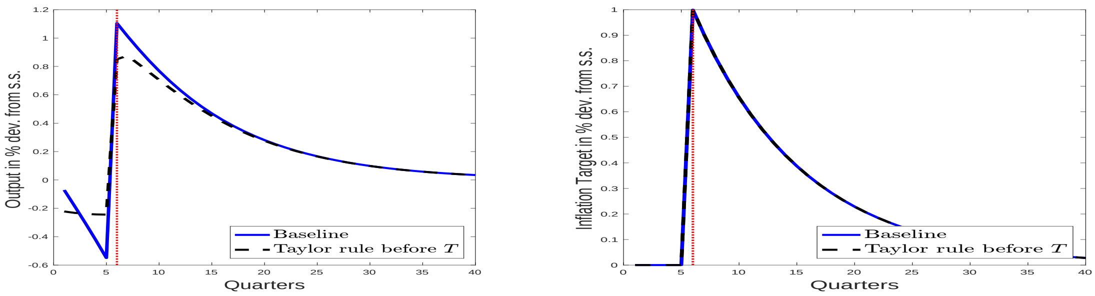


(a) Output Y

(b) Inflation Target Π^*

match the observed response of the yield curve. The new profile of the inflation target is shown in panel (b) of Figure A3, the response of output in panel (a). Output now falls immediately at the time of the announcement and remains at this lower level in all periods until T. This is due to the Taylor rule in (3) and the increased inflation in the interim period before T, which leads to an increase in nominal interest rates above their steady state level in all periods before T.

Figure A3: Response to Forward Guidance announcement with Taylor rule before T



(a) Output Y

(b) Inflation Target Π^*